

# NUMERICAL CHARACTERISATION OF NONLINEAR STIFFNESS PROPERTIES OF PRE-STRESSED VIBRATION ISOLATION MOUNTS

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## 1. INTRODUCTION

The VVES (Vibration of Viscoelastic and Elastic Structures) suite of programs were developed using a simplified vibration isolation model, in which the isolators were idealised by 2-noded flexible elements, which were connected to the upper and lower attachment points. Under the assumption of constant Poisson's ratio, the dynamic stiffness of each isolator can be represented by the product of complex frequency-dependent Young's modulus and a normalised 12x12 condensed stiffness matrix, which can be obtained through finite element analyses. The VVES program provides a linear finite element capability for this purpose. However, it can only handle vibration isolation mounts with relatively simple geometries under small deformation [1]. In other words, for practical mounts with complex geometries and/or subjected to finite static deformations, the condensed stiffness matrices must be pre-generated using more sophisticated external computational tools and imported to the VVES program. The purpose of the present study is to develop a numerical procedure for computing the condensed stiffness matrices for arbitrary vibration isolation mounts through nonlinear finite element analyses using the VAST program [2]. In this procedure, nonlinear finite element analysis is first performed up to the desired pre-stress level and the resulting tangent stiffness matrix is then used to compute the condensed matrix using a simple method based on prescribed unit displacements and rotations. The present procedure has been utilised to characterise three practical vibration isolation mounts, and for all of these mounts, good numerical results have been obtained.

## 2. CONDENSED STIFFNESS MATRIX

A typical vibration isolation mount contains top and bottom metal plating and a flexible visco-elastic core. All these structural components need to be included in the finite element analysis. In order to formulate the condensed stiffness matrix with respect to the attachment points, two additional nodes are introduced in the finite element model and multi-point constraint equations are utilised to enforce the displacement compatibility requirements between the attachment points and the surfaces of the top and bottom plates. Once the global stiffness matrix is computed, the

condensed stiffness matrix can be obtained by eliminating all the degrees-of-freedom from the system except those associated with the attachment points.

The elimination of these internal degrees-of-freedom can be achieved by using the method of static condensation and the condensed mass matrix can also be obtained by the method of Guyan reduction. However, this procedure involves a large number of matrix operations, which makes it less efficient computationally. For this reason, a simple method, which requires repeated solutions of the global finite element system subjected to prescribed unit displacements and rotations at the attachment points, has been utilised in the present work [3].

## 3. NONLINEAR FE FORMULATION

In a practical engine vibration isolation system, the isolators are normally subjected to static axial compression caused by the self-weight of the machinery before the application of the small amplitude dynamic load. This static compressive load is often large enough to produce finite deformation of the flexible mount. In order to include the effect of this initial finite deformation in the condensed stiffness matrix of isolators, the global tangent stiffness matrix at the full pre-loading must be employed in the calculation of the condensed stiffness matrix.

The computation of the tangent stiffness matrix requires nonlinear finite element analyses, which are performed using the general-purpose nonlinear finite element program, VAST. The VAST program provides a number of large strain, hyper-elastic solid elements, developed using the consistent co-rotational and total Lagrangian formulations [4]. A mixed method based on independent interpolation of pressure and displacement fields has been implemented to treat material incompressibility, and incompatible bubble modes have been employed to enhance behaviour of the lower-order element for bending dominated deformations. The hyper-elastic material property was represented using a revised version of Ogden's three-term strain energy function. This strain energy function is formulated in terms of principal stretches and includes 6 material constants in addition to the bulk modulus.

The large strain hyper-elastic capability in VAST has been extensively verified using a large number of test cases, including inflation of rubber cylinder and rectangular sheet and constrained compression of rubber blocks. For all these test examples, VAST results were compared with published analytical and numerical solutions. Close agreement has been obtained.

#### 4. APPLICATIONS

The VAST finite element program has been applied to predict condensed stiffness properties of three practical vibration isolation mounts, including the NETE mount, the PDE engine mount and the PDE raft mount [3]. The nonlinear behaviour associated with the pre-loading was modelled using the large strain hyper-elastic capability in VAST. All these mounts have complicated geometric shapes and are composed of top/bottom steel plating and a soft rubber core. Due to page limitation, only the results for the NETE mount are presented here.

The basic shape of the NETE mount is a nearly rectangular block. Each of the top and bottom steel plates contains four insertions into the rubber core material. All these insertions are of the same size and shape, which is a cylinder with a hemi-spherical top. In the present finite element analysis, a relatively coarse mesh has been utilised, in which the cylindrical insertions were approximated by rectangular ones, as shown in Figure 1. It is believed that this mesh is sufficient for characterising the overall stiffness property of the mount structure.

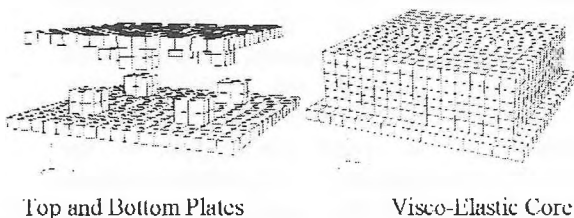


Fig. 1 Finite element model for nonlinear analysis of NETE mount.

After the construction of the finite element mesh, nonlinear material properties must be determined before finite element analysis. Because the top and bottom plates in this mount were both made of steel, the standard steel properties,  $E=2.07 \times 10^5 \text{ MPa}$  and  $\nu=0.3$ , were utilised. The selection of material constants for the hyper-elastic material was more difficult. In the present study, the six material constants required by the Ogden's model were first identified by curve-fitting the experimental data for uni-axial tension, and then scaled by matching the predicted and measured initial stiffness of the NETE mount for axial compression. Using these material properties, the VAST program predicted a nonlinear response, which is in close agreement with the experimental data for the complete loading range, as shown in Figure 2.

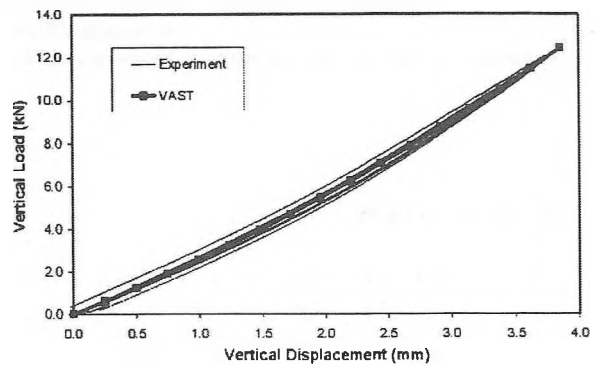


Fig. 2 Measured and predicted load-displacement curves.

After convergence was reached at each load step, the current tangent stiffness matrix was used for the computation of the  $12 \times 12$  condensed stiffness matrix. In order to investigate the variation of each element in the condensed stiffness matrix with the increase of pre-load, we obtained condensed matrices at a number of pre-load values. This investigation revealed that different elements in the condensed stiffness matrix vary with the magnitude of pre-load in very different ways. This is probably a consequence of the material anisotropy associated with the non-uniform stress distribution in the rubber component. As a result, the earlier method for taking into account the pre-load effect, which involves uniform scaling of the initial condensed stiffness matrix, is inaccurate and may result in errors in the dynamic simulation using the VVES programs.

#### 5. CONCLUSION

A computational procedure has been developed to evaluate the condensed stiffness matrix of vibration isolators required by the VVES program. This procedure is based on a nonlinear finite element program, VAST, which is able to handle complicated mount geometry and nonlinear effect associated with the static pre-loading. Three practical vibration isolation mount structures have been analysed and satisfactory numerical results have been obtained.

#### REFERENCES

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