REFLECTIONS ON BOUNDARIES

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Acousticians studying propagation in air and in water need to account for reflections at boundaries—both amplitude and phase effects—in order to model realistic scenarios. The typical assumptions made about boundaries in each discipline are somewhat different, however. This paper considers how acousticians in different disciplines cope with boundary effects and attempts to compare these methods.

A typical simplified boundary assumption for an acoustician working in air (either outdoors over ground or indoors with a porous wall covering) is the "locally reacting" boundary; that is, regardless of the incident field, the ratio between the pressure at the boundary and the normal component of the particle velocity is a single (although frequency-dependent) complex quantity called the surface impedance. In this case, the complex plane wave reflection coefficient $R(\theta)$ is related to the surface impedance z (normalized by the impedance of air) through the relation

$$R(\theta) = (z\sin\theta - 1)/(z\sin\theta + 1), \tag{1}$$

where θ is the grazing angle. Fig. 1 shows a calculation of the reflection loss $(-20\log|R(\theta)|)$ from such a surface for a typical case of normalized impedance $z = 2.5 - 2.5i$. Note the large values of reflection loss at all angles away from grazing incidence, the almost-linear relation between loss and angle at near-grazing incidence, and the maximum of reflection loss near 16 degrees.

One can invert Eqn. (1) to define an angle-dependent surface impedance in terms of a general reflection coefficient:

$$z(\theta) = \frac{1}{\sin\theta}\frac{1+R(\theta)}{1-R(\theta)}. \tag{2}$$

Using this relation, we can compare the "impedance" of a surface having a general reflection coefficient with a surface having a constant impedance.

Underwater acousticians commonly assume that the seabed can be modelled as a semi-infinite elastic solid, the reflection from which has been worked out by Brekhovsikh[1], among others. Consider the reflection from a "sand" layer having the following properties: density 1.8 (relative to water), compressional wave speed 1850 m/s, shear speed 300 m/s, compressional attenuation 0.46 dB/wavelength, and shear attenuation 0.23 dB/wavelength. Assuming a water sound speed of 1500 m/s, the reflection coefficient can be calculated using the standard formula, and from Eqn. (2) the effective impedance follows. Fig. 2 shows the angle-dependent impedance for the sand layer and Fig. 3 shows the reflection loss.

In contrast to the locally-reacting (constant impedance) surface, the impedance of the visco-elastic solid shows a strong dependence upon the grazing angle of the incident plane wave, but this divides naturally into three regions: at low grazing angles the impedance is nearly constant, mostly negative imaginary (reactive); in the region of the critical angle for transmission of compressional waves into the seabed, the impedance changes rapidly from imaginary to real; at large grazing angles the impedance is again nearly constant, but is now positive real (resistive). The reflection loss curve also shows a large change at the critical angle. Note (again) the almost-linear relation between loss and angle at near-grazing angles.

The simplest boundary reflection models used by air acousticians (on the one hand) and underwater acousticians (on the other hand) have quite different characteristics; however, they are similar at near-grazing incidence, in that they both show near-linear relations between reflection loss and angle. In other words, in both cases the impedance is nearly constant at near-grazing angles.

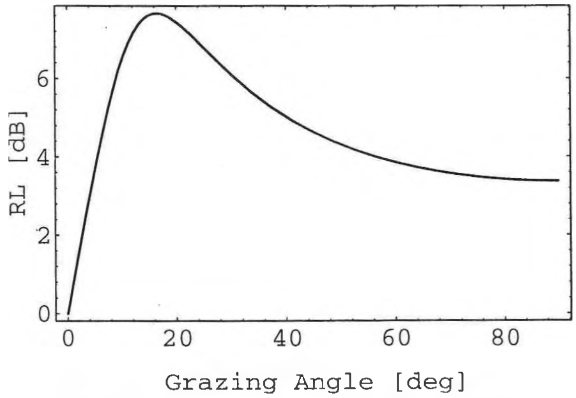

Figure 1 *Reflection loss (RL) of a locally-reacting surface with normalized impedance 2.5 2.5i.*

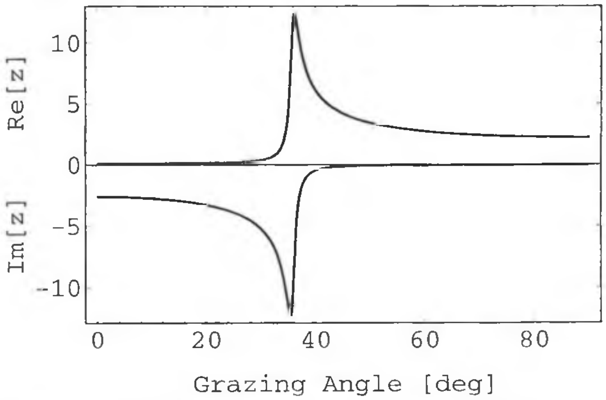

Figure 2 *Real (+) and imaginary (–) components of the normalized surface impedance (z) of the sand layer.*

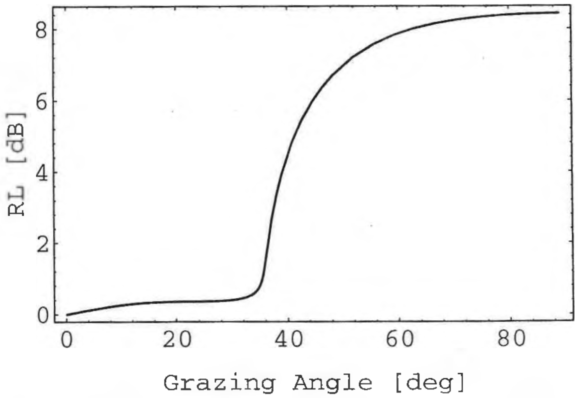

Figure 3 *Reflection loss (RL) of a sand layer, calculated using a visco-elastic solid model.*

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[1] L.M. Brekhovskikh, *Waves in Layered Media* (Academic, New York, 1980), 2nd ed.