1 INTRODUCTION

Numerical predictions based on the parabolic equation (PE) approximation are routinely used to model sound propagation in air and underwater. The main rationale for this is that accurate full-wave solutions to the PE can be computed efficiently using marching algorithms for both depth- and range-dependent inhomogeneous media. The development of the PE method has reached the point where finite-difference implementations derived from Padé series expansions can provide accurate solutions to one-way wave propagation for realistic geoacoustic conditions, e.g., over variable-depth bathymetry in the sea or variable-elevation topography in air. Moreover, with the introduction of both exact and approximate PE procedures for handling elastic media and rough-surface boundaries, the physics of shear wave propagation and forward-scattering can readily be accommodated.

2 THEORY

In two dimensions \((r, z)\), \((z\) positive down\), the outgoing spatial component of the acoustic pressure \(p \exp(-i \omega t)\) can be recovered from the field \(\psi = p \exp(-ik_0r)\sqrt{k_0r}\) that satisfies the higher-order Padé PE [1]

\[
\frac{\partial \psi}{\partial r} = ik_0 \sum_{j=1}^{J} \frac{a_{j,j}(\epsilon + \mu)}{1 + b_{j,j}(\epsilon + \mu)} \psi.
\]

(1)

Here \(\epsilon = N^2 - 1, \mu = k_0^{-2} \rho \partial_2 (\rho^{-1} \partial_2), k_0 = \omega/c_0, N = n(1 + i\alpha), n = c_0/c\) and \(\rho, c\) and \(\alpha\) denote the density, sound speed and absorption, respectively. Although real-valued Padé coefficients \(a_{j,j}\) and \(b_{j,j}\) are known in analytical form [1], it is convenient for some applications to use complex-valued coefficients which must be determined numerically [2]. Using the method of fractional steps and the Crank-Nicolson finite-difference procedure, Eq. (1) is efficiently solved at each range step \(\Delta r\) as a sequence of \(J\) systems of tri-diagonal equations.

3 EXAMPLE

To illustrate the capability of Eq. (1), we consider the deterministic rough-surface test case examined at a recent Reverberation and Scattering Workshop [3]. Instead of forcing the PE to accommodate a non-flat pressure-release boundary, we modified the original problem by appending an air-layer backing to the region above the rough surface. By this maneuver, scattering by an external pressure-release boundary was replaced with scattering by an internal fluid/fluid interface across which the usual boundary conditions on the acoustic field apply. The large impedance drop across the ocean/air interface \((\approx 2 \cdot 10^{-4})\) results in nearly perfect, out-of-phase reflection of sound for a water-borne source. A gaussian-tapered beam \((f = 400\) Hz\) was steered upwards toward the surface at an angle of \(10^\circ\) to the horizontal. The full-field result for \(|p|\) obtained using Eq. (1) is shown in Fig. 1 for a 20-m air layer backing and \(J = 2\). The rough surface clearly scatters sound to steeper angles. This forward-scattered PE result agrees almost exactly with a reference solution obtained using an integral equation method [3].

REFERENCES

