PE-ING IN AIR AND UNDERWATER

David J. Thomson

Defence Research Establishment Atlantic
Esquimalt Defence Research Detachment, FMO Victoria, B.C. V0S 1B0

1 INTRODUCTION

Numerical predictions based on the parabolic equation (PE) approximation are routinely used to model sound propagation in air and underwater. The main rationale for this is that accurate full-wave solutions to the PE can be computed efficiently using marching algorithms for both depth- and range-dependent inhomogeneous media. The development of the PE method has reached the point where finite-difference implementations derived from Padé series expansions can provide accurate solutions to one-way wave propagation for realistic geoacoustic conditions, e.g., over variable-depth bathymetry in the sea or variable-elevation topography in air. Moreover, with the introduction of both exact and approximate PE procedures for handling elastic media and rough-surface boundaries, the physics of shear wave propagation and forward-scattering can readily be accommodated.

2 THEORY

In two dimensions (r, z), (z positive down), the outgoing spatial component of the acoustic pressure $p\exp(-i\omega t)$ can be recovered from the field $\psi = p\exp(-ik_0 r)\sqrt{k_0 r}$ that satisfies the higher-order Padé PE [1]

$$\frac{\partial\psi}{\partial r} = ik_0 \sum_{j=1}^{J} \frac{a_{j,J}(\varepsilon+\mu)}{1+b_{j,J}(\varepsilon+\mu)}\psi. \tag{1}$$

Here $\varepsilon = N^2 - 1$, $\mu = k_0^{-2}\rho\partial_z\left(\rho^{-1}\partial_z\right)$, $k_0 = \omega/c_0$, $N = n(1+i\alpha)$, $n = c_0/c$ and ρ, c and α denote the density, sound speed and absorption, respectively. Although real-valued Padé coefficients $a_{j,J}$ and $b_{j,J}$ are known in analytical form [1], it is convenient for some applications to use complex-valued coefficients which must be determined numerically [2]. Using the method of fractional steps and the Crank-Nicolson finite-difference procedure, Eq. (1) is efficiently solved at each range step Δr as a sequence of J systems of tri-diagonal equations.

3 EXAMPLE

To illustrate the capability of Eq. (1), we consider the deterministic rough-surface test case examined at a recent Reverberation and Scattering Workshop [3]. Instead of forcing the PE to accommodate a non-flat pressure-release boundary, we modified the original problem by appending an air-layer backing to the region above the rough surface. By this maneuver, scattering by an external pressure-release boundary was replaced with scattering by an internal fluid/fluid interface across which the usual boundary conditions on the acoustic field apply. The large impedance drop across the ocean/air interface ($\approx 2\cdot 10^{-4}$) results in nearly perfect, out-of-phase reflection of sound for a water-borne source. A gaussian-tapered beam (f = 400 Hz) was steered upwards toward the surface at an angle of 10° to the horizontal. The full-field result for $|p|$ obtained using Eq. (1) is shown in Fig. 1 for a 20-m air layer backing and $J = 2$. The rough surface clearly scatters sound to steeper angles. This forward-scattered PE result agrees almost exactly with a reference solution obtained using an integral equation method [3].

REFERENCES

[1] F.B. Jensen, W.A. Kuperman, M.B. Porter and H. Schmidt, *Computational Ocean Acoustics*, (AIP Press, New York, 1994), Ch. 6, pp. 343–412.

[2] M.D. Collins, "A two-way parabolic equation for elastic media," J. Acoust. Soc. Am. **93**, 1815–1825 (1993).

[3] D.J. Thomson, G.H. Brooke and E.S. Holmes, "PE approximations for scattering from a rough surface," Defence Research Establishment Pacific, Victoria, B.C., Tech. Memo. 95–21, March 1995.

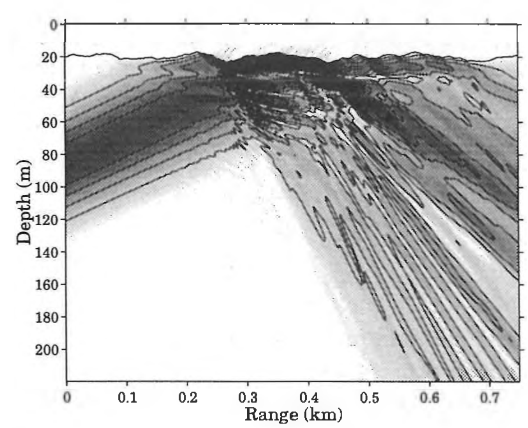

Figure 1: Forward scattering from a rough surface.