A Bernoulli-Euler Stiffness Matrix Approach for Vibrational Analysis of Linearly Tapered Beams

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1. Introduction. A particular area of interest is the vibrational behaviour of blade-type structures in which tapered beams have been, widely, studied. In finite element formulations[1], polynomial shape functions are often used. A deviation from this practice will pay dividends if improved accuracies can be obtained by using other shape functions . This is the case of the Dynamic Stiffness Matrix (DSM) appoach, in which, the frequency dependent shape functions are used. In this paper an appoach leading to the DSM of Bernoulli-Euler linearly tapered beams is presented.

2.Method. The differential equation for the lateral free vibrations of a Bernoulli-Euler tapered beam is:

$$(H_{fy}(x)w_{,xx})_{,xx} + m(x)w_{,tt} = 0. \quad (1)$$

where, $H_{fy}(x) = EI_y(x)$, $m(x) = \rho A(x)$; w and I_y represent displacement and the second moment of area. Considering harmonic vibrations, eq.(1) becomes;

$$H_{fy}(w_{,xxxx} - \alpha^4 w) - (\underbrace{H_{fyDEV}}_{(H_{fy} - H_{fy}(x))} w_{,xx})_{,xx} + \underbrace{m_{DEV}}_{(m - m(x))} \omega^2 w = 0. \quad (2)$$

where; $\alpha^4 = m\omega^2/H_{fy}; H_{fy} = H_{fy}(x)_{average}; m = m(x)_{average}$.

The non-dimensionalized weak form for the element k, associated to eq.(2) can be written as:

$$W^k{}_{ND} = \int_0^1 (\frac{\gamma_k}{\bar{l}_k^3} \bar{w}'' \bar{\delta w}'' - \mu^2 \bar{m}_k \bar{l}_k \bar{w} \bar{\delta w}) d\eta_k + DEV. \ ; \quad (3)$$

$$DEV. = -(\frac{1}{\bar{l}_k^3}) \int_0^1 (\gamma_{DEV} \bar{w}'' \bar{\delta w}'') d\eta_k + (\mu^2 \bar{l}_k) \int_0^1 (\bar{m}_{DEV} \bar{w} \bar{\delta w}) d\eta_k$$

where; $\gamma_k = EI_k/EI_r$; $\bar{l}_k = l_k/L$; $\mu^2 = m_r \omega^2 L^4/EI_r$;

$\bar{m}_{DEV} = m_{DEV}/m_r$; $(r \equiv reference\ value)$.

Eq.(3) is written in the equivalent form:

$$W^k{}_{ND} = \frac{\gamma_k}{\bar{l}_k^3} \int_0^1 \underbrace{(\bar{\delta w}'''' - \bar{\alpha}^4 \bar{\delta w})}_{(*)} \bar{w} d\eta_k + \frac{\gamma_k}{\bar{l}_k^3} [\bar{\delta w}'' \bar{w}' - \bar{\delta w}''' \bar{w}]_0^1 + DEV. \quad (4)$$

Then, $\bar{\delta w}$ and $\bar{w}$ are approximated so that (*) vanishes:

$$\bar{\delta w} = < P(\eta) > \{\bar{\delta a}\}; \bar{w} = < P(\eta) > \{\bar{a}\} \quad (5)$$

Considering the four nodal variables: $\bar{w}_1; \bar{w}_1'; \bar{w}_2; \bar{w}_2'$ as $< \bar{\delta a} >$, we obtain $\{\bar{\delta w}_n\} = [P_n] * \{\bar{\delta a}\}$ and hence, the approximation(5) in nodal variables is written as:

$$\bar{w}(\eta) = < P(\eta) > [P_n]^{-1} \{\bar{w}_n\} = < N(\eta) > \{\bar{w}_n\} \quad (6)$$

and by using eq.(4) the element DSM is obtained as:

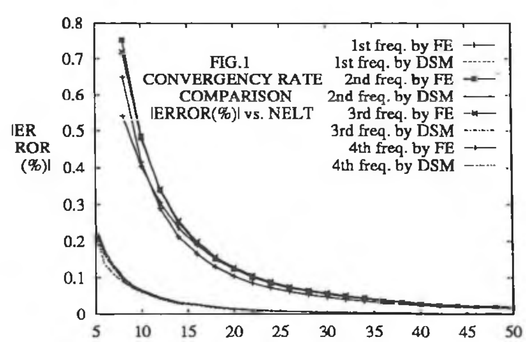
$$W^k{}_{ND} = < \bar{\delta w}_n > \underbrace{([K_{RD}]_{uni} + [K_{RD}]_{DEV})}_{[K_{RD}]} \{\bar{w}_n\}; \quad (7)$$

where; $[K_{RD}]_{uni} = \frac{\gamma_k}{\bar{l}_k^3} [\{N'''\}_0 \{-N''\}_0 \{-N'''\}_1 \{N''\}_1]$;

and $[K_{RDij}]_{DEV} = -(\frac{1}{\bar{l}_k^3}) \int_0^1 \gamma_{DEV} N_i'' N_j'' d\eta_k + (\mu^2 \bar{l}_k) \int_0^1 \bar{m}_{DEV} N_i N_j d\eta_k$.

Elementary matrices are assembled, and the bisection method[2] is used to find natural frequencies of beams.

3.Results. The DSM is used to find natural frequencies of a cantilever tapered beam so $A_L = \frac{A_0}{2}$(FIG.1), and the results have been compared to those found by F.E. Much better convergency rate is found by DSM.



References: [1]Dhatt G.and TouzotG.- Finit Element Method Displayed. [2]Wittrick W.H.and Williams F.W. Int.J.mech.Sci. 1970. Vol.12, pp781-791.