Nonlocal Tops and Equivalent Bottoms for Sound Propagation in Air Problems

David J. Thomson
Defence Research Establishment Atlantic
P.O. Box 1012, Dartmouth, Nova Scotia, Canada, B2Y 3Z7

INTRODUCTION

One-way wave equations derived from parabolic equation (PE) approximations are widely used to model underwater and atmospheric sound propagation (see [1, Chap. 6], [2] and the references therein). Finite-difference PE solvers based on Padé series expansions provide accurate and efficient solutions to these one-way fields for range-varying geoacoustic environments [3]. For layered media, alternative approaches based on normal mode, multipath expansion or wavenumber integration representations are available [1], [4]. Since proper analysis of acoustic field behaviour often relies on using more than one propagation model, it is desirable to obtain numerical agreement between models in situations where different models apply.

To solve a PE numerically, the computational grid must be terminated top and bottom. In outdoor sound applications, the acoustic field is usually assumed to satisfy a locally-reacting (constant impedance) boundary condition along the ground plane [2], [4]. This condition is easily incorporated into finite-difference PE models [5]. On the other hand, wavenumber integration codes such as SAFARI, that were developed specifically for underwater sound propagation applications, do not incorporate this locally-reacting condition directly [6], [7]. In the first part of this paper, we design an equivalent fluid whose reflection response is numerically equivalent to that produced by a constant impedance surface.

Wavenumber integration models inherently satisfy a radiation condition as $z \to \infty$. In contrast, PE solvers usually handle upgoing waves by appending an artificial absorbing layer to the computational mesh in order to attenuate the radiated energy. In the second part of this paper, we present a nonlocal boundary condition (NLBC) that exactly transforms the semi-infinite PE problem with a radiation condition at $z \to \infty$ to an equivalent PE problem in a bounded domain [8], [9], [10], [11].

We provide a numerical example that compares SAFARI predictions (obtained with an equivalent bottom) to PE predictions (obtained with an NLBC top) for a problem that typifies outdoor sound propagation.

PE BASICS

For sound propagation in 2D (range r, height z), the outgoing spatial component $p(r,z)$ of the acoustic pressure $p\exp(-i\omega t)$ can be recovered from the reduced field $\psi = p\exp(-ik_0 r)\sqrt{k_0 r}$, where $k_0 = \omega/c_0$, by solving

$$\frac{\partial \psi}{\partial r} = ik_0\left(-1+\sqrt{1+X}\right)\psi. \qquad (1)$$

Here $X = N^2 - 1 + k_0^{-2}\rho\partial_z\left(\rho^{-1}\partial_z\right)$, $N = n(1+i\alpha)$, $n = c_0/c$ and ρ, c and α denote the density, sound speed and absorption, respectively. The field ψ also satisfies a radiation condition as $z \to \infty$. Setting $\delta = k_0\Delta r$, the formal solution to (1) is given by

$$\psi(r+\Delta r, z) = \exp\left(-i\delta + i\delta\sqrt{1+X}\right)\psi(r,z) \ . \qquad (2)$$

One higher-order procedure for solving (2) involves expanding the square-root operator in the Padé series [12]

$$-1+\sqrt{1+X} \approx \sum_{m=1}^{M}\frac{a_m X}{1+b_m X} , \qquad (3)$$

so that (2) can be cast in the form

$$\psi(r+\Delta r, z) = \prod_{m=1}^{M}\exp\left(\frac{i\delta a_m X}{1+b_m X}\right)\psi(r,z) \ . \qquad (4)$$

For sufficiently small δ, each propagator can be accurately approximated by its unitary Cayley form and (4) can be solved recursively for $m = 1,\ldots,M$ as

$$\left(1+c_m^- X\right)\psi_m(r,z) = \left(1+c_m^+ X\right)\psi_{m-1}(r,z) , \qquad (5)$$

where $c_m^\pm = b_m \pm \frac{1}{2}i\delta a_m$, $\psi_0(r,z) \equiv \psi(r,z)$ and $\psi_M(r,z) \equiv \psi(r+\Delta r, z)$. The operator X is handled numerically using a three-term finite-difference approximation so that each system in (5) is tridiagonal. This procedure advances the PE field one range step. In this paper, we limit our discussion to the single-term PE that results when $M = 1$, $a_1 = \frac{1}{2}$, and $b_1 = \frac{1}{4}$.

EQUIVALENT BOTTOM

The reflection coefficient associated with a locally-reacting boundary is given by (θ is the grazing angle)

$$R'(\theta) = \frac{Z'\sin\theta - 1}{Z'\sin\theta + 1} , \qquad (6)$$

where $Z' = X + iY$ is the ground impedance normalized by $Z_a = \rho_a c_a$, the impedance of air. In contrast, the reflection coefficient due to a uniform half-space can be written as

$$R(\theta) = \frac{(Z_g/Z_a)\sin\theta - \sqrt{1-\cos^2\theta/n_g^2}}{(Z_g/Z_a)\sin\theta + \sqrt{1-\cos^2\theta/n_g^2}}, \qquad (7)$$

where $Z_g = \rho_g c_g$ and $n_g = (c_a/c_g)(1+i\alpha_g)$ are the impedance and refractive index of the lossy ground, respectively. The goal is to choose c_g, ρ_g, and α_g to make $R = R'$ for all θ. Although this can't be done exactly, we can satisfy $Z_g/Z_a = Z'$ approximately by choosing c_g so that $n_g^2 \gg 1$ and solving for ρ_g and α_g from

$$\frac{(\rho_g c_g)/(\rho_a c_a)}{1+i\alpha_g} = X + iY , \qquad (8)$$

to determine the parameters of the equivalent fluid [10].

NONLOCAL TOP

PE calculations of sound propagation in air usually approximate the radiation condition as $z \to \infty$ by appending an absorbing layer to the top of the computational grid and setting the field to zero at the top of the absorber [1], [2]. Here, we introduce a nonlocal boundary condition (NLBC) that can be applied at a finite height $z = h$ and that does not require an absorbing layer. The medium in $z > h$ is assumed to be uniform. For the first-order PE corresponding to $M = 1$, an NLBC can be derived in the form [11]

$$\left\{\frac{\partial}{\partial z} + i\,(\rho_a/\rho_g)\,\Gamma_1\right\}\psi(r+\Delta r, h) = 0, \quad (9)$$

where Γ_1 is the vertical wavenumber operator defined by

$$\Gamma_1^2 = k_0^2\left(N_g^2 - 1 + \frac{4i}{\delta}\frac{1-\mathcal{R}}{1+\mathcal{R}}\left[1+\frac{i}{\delta}\frac{1-\mathcal{R}}{1+\mathcal{R}}\right]^{-1}\right), \quad (10)$$

and $\mathcal{R} = \exp(-\Delta r \partial_r)$ is a range translation operator. Noting that $\mathcal{R}^j\psi(r,h) = \psi(r - j\Delta r, h)$, Γ_1 can be expanded in a Taylor series in $\mathcal{R}$ to yield a nonlocal implementation of (9), i.e., the field $\psi(r + \Delta r, h)$ is expressed in terms of the known field along $0 \to r$. Nonlocal boundary conditions derived from spectral formulations are considered elsewhere [8], [9], [10].

EXAMPLE

Consider the sound speed profile $c(z) = 330 + 0.12z$ m s^{-1} for $0 < z < 100$ m capped by a uniform half-space of speed 342 m s^{-1} [5]. The air in $z > 0$ is taken to have uniform density 0.0012 g cm^{-3} and absorption 0 dB λ^{-1}. Calculations are carried out for a 40-Hz source at $z = 2$ m and a receiver along $z = 1$ m above an impedance plane where $Z' = 31.4 - 38.5i$. Choosing $c_g = 33$ m s^{-1} in (8) yields $\rho_g = 0.9422$ g cm^{-3} and $\alpha_g = 66.92$ dB λ^{-1} for the equivalent fluid for use with SAFARI. The NLBC in (9) for use with the PE model was applied along the top of the refracting layer, $z = 100$ m. The PE calculations were carried out using $\Delta r = 1$ m, $\Delta z = 0.25$ m and $c_0 = 330$ m s^{-1}. SAFARI and PE predictions of transmission loss ($-10\log_{10}|p|^2$) versus range are compared in Fig. (1). The agreement between the two model predictions is observed to be excellent. For this downward refracting profile, several trapped modes are observed to interfere coherently as a function of range.

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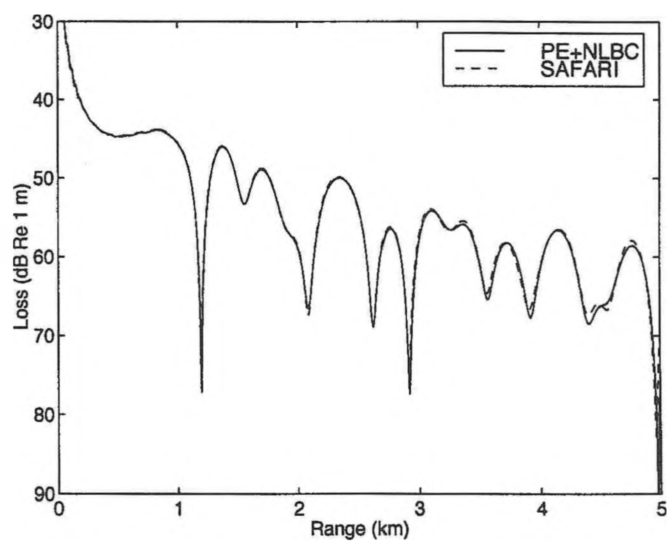

Figure 1: Transmission loss comparison.