Simulations Of Full-Field Tomography Of Oceanic Currents In Shallow Areas

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Matched Non-reciprocity Tomography (MNT)

According to the acoustical reciprocity principle, the CW acoustic pressures measured in reciprocal transmissions scheme are identical in motionless stationary media [1]. Oceanic currents are relatively slow, but even slow flows break reciprocity. Observing reciprocity breaking effects seems to be the only reliable way of detecting currents by acoustical methods. Usually acoustic tomography of currents relies on measuring non-reciprocity of travel times along acoustic rays. The approach experiences difficulties in shallow water due to problems with ray resolution and identification [2].

MNT [3,4] is an alternative full-field technique that generalizes Matched Field Processing [5] for the problem of flow monitoring. MFP solves inverse problems through multiple solutions of the forward problem. First, the sound propagation is simulated numerically for a large set of possible models of the environment. Then the simulation results are compared to the experimental data using some criterion. The model that provides the best match is taken as a solution of the inverse problem.

To detect currents MNT compares not the acoustic fields, but the differences in some acoustical quantity measured (and simulated) in reciprocal transmissions. In the absence of flows, there is no difference between reciprocal data and the method correctly gives zero current. Any non-reciprocity is transformed into flow field.

Put into practice, this simple idea leads to new problems:

- the computer model must correctly describe acoustical effects of oceanic currents in complicated shallow-water environments
- the method for comparing experimental and numerical results should be sensitive to flows, but not sensitive to uncertainties in other environmental parameters, like bottom topography and sound speed field
- the method is computationally demanding

These issues and possible solutions are addressed in the following sections.

Direct problem

In theory, acoustical fields in motionless and stationary media are symmetric with respect to interchange in source and receiver positions. However, most of numerical models do not preserve this fundamental property for general range-dependent media. (Ray models are a notable exception.) The fields predicted for reciprocal transmissions are different. The discrepancy is often related to bottom topography and sound speed field. For complicated scenarios it becomes comparable or exceeds the effects of flows. MNT inversion with such propagation models will attribute this non-reciprocity to currents and will give erroneous results.

An energy-conserving and reciprocal One Way Wave Equation (OWWE) for motionless fluid was proposed in [6]. For moving media, a closely related Generalized Claerbout PE (GCPE) was derived [7]. Application of OWWE to moving media is discussed in [8]. Efficient algorithm for solving GCPE and OWWE is developed in [8, 9]. The IFD scheme exactly comply with the reciprocity principle and the flow reversal theorem (FRT), which is an extension of reciprocity principle to moving media [1].

To demonstrate GCPE properties consider propagation of sound over a rugged wedge shown in Fig. 1. Sound speed field is defined by three vertical profiles at 3, 18, and 33 km. The profiles (not shown) approximately correspond to conditions in the Strait of Florida in summer. Point transceivers are located at 200 m depth 36 km apart. CW frequency is 50 Hz.

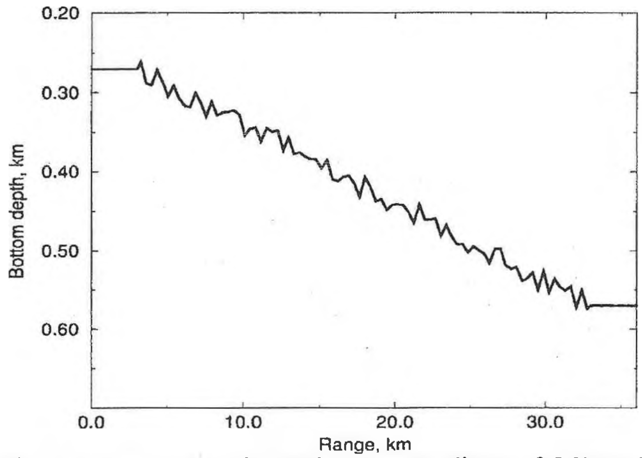

Fig. 1. Bottom topography used to test compliance of GCPE with the acoustical reciprocity principle

The acoustical amplitudes and phases predicted by GCPE for up-slope and down-slope propagation are presented in Fig. 2. The last 40 m of the propagation paths are shown. The fields at the transceivers' locations are identical. The agreement is within the limits of round-off errors: about 10^{-10} dB in amplitude and 10^{-10} degrees in phase.

Complicated bottom topography strongly affects acoustic fields. Acoustical imprints of currents are much smaller. However, the topography effects cancel each other in reciprocal transmissions. The subtle effects of currents are isolated in non-reciprocity.

The developed OWWE and GCPE models are adequate for acoustic monitoring of flows.

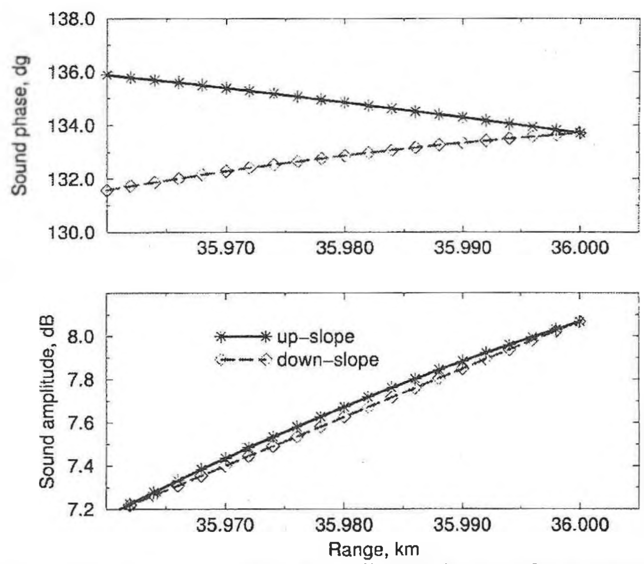

Fig. 2. Sound phases (top) and amplitudes (bottom) for reciprocal transmissions over the wedge of Fig. 1.

Cost functions

Theoretical and experimental non-reciprocities are compared using a measure or norm that in MFP and MNT is referred to as ***cost function***. The simplest cost function is RMS difference of non-reciprocities of complex acoustic pressure. However, this trivial cost function has poor performance [4, 10].

In real experiment, the exact geometry, bottom topography and properties, etc., are never known. Non-reciprocities of various acoustical quantities have different sensitivities to environmental parameters. Corresponding cost functions will have different response to experimental errors.

Theoretical analysis and numerical simulations in [3, 4, 10] revealed that non-reciprocity of acoustical phase does not depend on small variations in sound speed, density, bottom topography and geophysical properties, and horizontal separation between the transceivers. It is still quite sensitive to the vertical distribution of the flow field. On the opposite, non-reciprocity of complex pressure is as sensitive to the above mismatches as the one-way sound field itself. Non-reciprocity of acoustic amplitude reveals only weak dependence on the vertical profile of the flow. Robust cost functions should compare phase non-reciprocities. Auxiliary amplitude information might be used to give larger weight to phase data with good signal-to-noise ratio.

Originally, MNT was formulated for CW signals in accordance with MFP [4]. For CW sound, it is important to collect the data at many depths. A cost function using CW data from a single depth would not provide much resolution. Measuring depth dependence of non-reciprocity requires a vertical array of transceivers. This can be a synthetic aperture array, as MNT in not sensitive to mismatches in propagation range.

Acoustical phase is known with uncertainly of the total number of periods. RMS difference of phases becomes unstable when any phase is close to the limits of $[-\pi,\pi]$ period. A robust cost function is constructed from sine and cosine components of phase difference. A typical MNT cost function is [10]

$$F_{MNT} = 2\left(1-\left[\langle I\cos\Delta\vartheta\rangle^2+\langle I\sin\Delta\vartheta\rangle^2\right]^{1/2}\Big/\langle I\rangle\right). \qquad (1)$$

Here $I=\left|p_e^{(+)}p_e^{(-)}p_t^{(+)}p_t^{(-)}\right|^{1/2}$ and $\Delta\upsilon=\arg\left(p_e^{(+)}\tilde{p}_e^{(-)}\tilde{p}_t^{(+)}p_t^{(-)}\right)$ are the amplitude weighting multiplier and phase non-reciprocity. Acoustic pressure fields $p^{(\pm)}$ correspond to downstream and upstream propagation; tilde denotes complex conjugation. Indexes "e" and "t" refer to experimental and predicted values. Angular brackets stand for averaging over a transceiver array.

Phase non-reciprocity is a distinct and non-degenerate function of frequency [10, 11]. Therefore, for current velocity inversions, the data on the depth-dependence of phase non-reciprocity can be efficiently combined with its frequency dependence. For realistic problems considered in [10, 11] the inversion was robust with only 3-4 transceivers. Hence, MNT can be implemented using standard tomographic transceivers instead of specialized transceiver arrays. Cost function for multi-frequency MNT is obtained, e.g., by averaging (1) over frequency.

Linearization

Search of the global minimum of MNT cost function requires repeated calculations of sound fields in range-dependent ocean for many times. The number of iterations rapidly increases with dimension of $\mathbf{q}$ space. The problem is strongly non-linear.

To accelerate inversion two linearized methods were developed [12]. For MFP linearization is possible for variations of the sound speed field as small as 1-2 m/s [5]. For temperature tomography this restriction is usually unacceptable. On the opposite, the amplitude of current velocity in the ocean is usually within these limits and linearization looks promising. Moreover, the second order terms mutually cancel in reciprocal values to be used for inversion which favors to linearization.

Non-reciprocity of acoustic phase depends linearly on the parameters of the flow model within some interval of their variation

$$\vartheta_t(\mathbf{q}) \approx \vartheta_t(\mathbf{q}_0) + \nabla\vartheta_t(\mathbf{q}_0)\cdot(\mathbf{q}-\mathbf{q}_0)\,. \qquad (2)$$

The first approach called complete linearization is to substitute (2) in (1) and convert F_{MNT} into a quadratic form of $\mathbf{q}-\mathbf{q}_0$. Good initial estimate $\mathbf{q}_0$ is required. In this approximation the full field phase non-reciprocity measured as a function of depth or frequency is converted into the flow field using existing linear inversion methods. These methods intrinsically provide the estimates of uncertainty of the inversion results.

The second approach is closely related to the normal mode theory. The phases of separate normal modes are calculated with (2) and then substituted into the standard modal formulas for $p_t^{(\pm)}(\mathbf{q})$. The cost function (1) remains non-linear with respect to $\mathbf{q}-\mathbf{q}_0$. However, its computation is much faster. This approach is referred to as partial linearization. Partial and complete linearization can be combined in one scheme.

Summary

The presented methods are essential components of acoustical toolbox for fast, accurate, and robust reconstruction of oceanic currents in shallow water environment. Their capabilities are confirmed by numerical simulations.

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