SIMULATION OF UNDERWATER AMBIENT NOISE TIME SERIES

Daniel Hutt, Paul C. Hines and Andrew Rosenfeld
Defence Research Establishment Atlantic
P.O. Box 1012, Dartmouth, Nova Scotia, Canada, B2Y 3Z7

In modern naval surveillance, ships and submarines may be tracked via their underwater acoustic signatures. In both active and passive sonar, arrays of hydrophones are used to enhance the signal-to-noise ratio and to obtain directional information. Various techniques exist to process the acoustical signals from arrays in order to maximize the sensitivity in a desired direction while minimizing the contribution of ambient noise. Two array processing techniques currently being explored at Defense Research Establishment Atlantic (DREA) are superdirective and intensity processing. In order to test these concepts for underwater applications, it is necessary to simulate the response of an array to an acoustic signal in the presence of ambient noise. The results obtained from the signal processing algorithms will be affected by the spatial and temporal characteristics of the noise field. Since real acoustic data may be unavailable or the statistics may not be fully known, simulated noise can be used to probe array performance as a function of quantifiable noise characteristics.

In this paper we present three different approaches for generating synthetic ambient noise time series data which possess controlled statistical characteristics. The noise statistics which are specified are the probability density function, power spectrum, and the complex cross correlation function between pairs of noise time series. The simulated noise time series represent different types of underwater noise fields.

1. Synthetic noise time series for a single hydrophone

To create synthetic ambient noise time series from a single hydrophone, we used the approach of Walker (Ref. 1) where a one dimensional autoregressive moving average (ARMA) filter is applied to discrete Gaussian white noise. ARMA filters, also known as finite impulse response filters, are designed to have frequency responses that approximate the power spectra of different undersea noise conditions. The filtered Gaussian noise yields a time series with the desired spectral properties. ARMA filters have been used successfully to simulate time series representing a wide range of processes from car traffic noise to financial markets (Ref. 2).

ARMA filters have the form

$$y_k = \sum_{i=1}^{m} a_i y_{k-i} + b_0 x_k \ , \qquad (1)$$

where, x_k is the input white noise value at time sample k and y_k is the k^{th} filter output. y_k is determined by previous values of y through the m coefficients a_1, a_2,...a_m. The quantity b_0^2 is known as the prediction error variance and is a measure of how well the power spectrum of y matches the spectrum being modelled.

To create a filter, the desired noise frequency spectrum is Fourier transformed to obtain the autocovariance function with elements R_j where j refers to the jth time lag. The filter coefficients a_i are found by solving the equation:

$$R_j = \sum_{i=1}^{m} a_i R_{j-i} \qquad 1 \le j \le m. \qquad (2)$$

Equation 2 defines a set of m simultaneous equations in m unknowns. In matrix form it may be expressed as

$$\begin{bmatrix} R_0 & R_{-1} & R_{-2} & \dots & R_{1-m} \\ R_1 & R_0 & R_{-1} & \dots & R_{2-m} \\ : & : & : & : & : \\ R_{m-1} & R_{m-2} & R_{m-3} & \dots & R_0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ : \\ a_m \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ : \\ R_m \end{bmatrix} . \qquad (3)$$

The m-order system of Eq. 3 can be solved with approximately m^3 operations using Gaussian elimination. However, the matrix **R** has a Toeplitz structure which can be inverted with approximately m^2 operations using the Levinson recursion algorithm (Ref. 3, p. 359-367).

An example of an ambient noise power spectrum is shown in Fig. 1. It is based on the empirical noise model of Merklinger and Stockhausen (Ref. 4) which combines estimated contributions of noise from shipping, surface noise caused by wind and the intrinsic noise of the recording system. The parameters used in the Merklinger and Stockhausen model are wind speed of 40 km/hr and a moderate shipping noise level of 86 dB//μPa²/Hz. A filter designed to model this noise spectrum was applied to a Gaussian white noise time series resulting in a time series with the power spectrum shown in Fig. 1. Clearly the power spectrum of the simulated noise time series closely resembles the power spectrum being modelled.

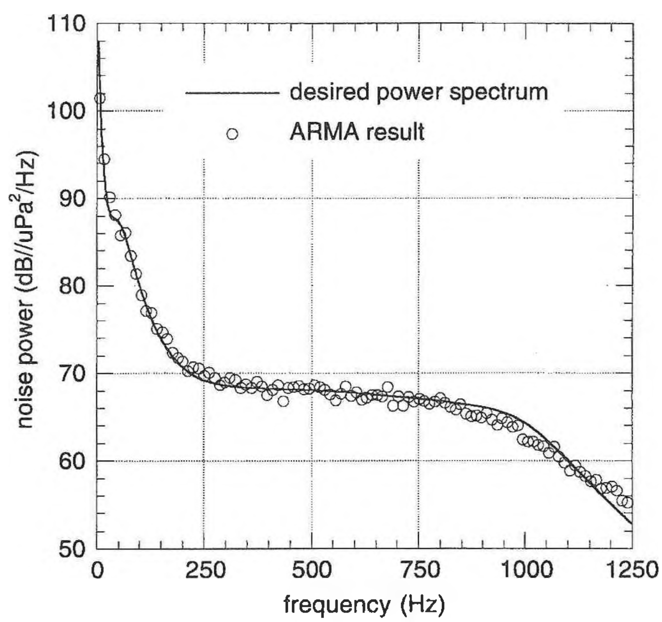

Fig. 1 Example power spectrum from the model of Ref. 4 and power spectrum of simulated noise using an ARMA filter.

2. Synthetic noise time series for a hydrophone array

Although ARMA filters are widely used for time series simulations, they are only appropriate for generating ambient noise for a single hydrophone. For realistic simulation of noise signals from an array of hydrophones, the generated signals must possess the appropriate cross-correlation. The degree of correlation of noise signals $s_1(t)$ and $s_2(t)$ measured at two hydrophones is characterized by the normalized cross-spectrum $S_{1,2}(\omega)$,defined as

$$S_{1,2}^2(\omega) = \frac{S_1(\omega)S_2^*(\omega)}{|S_1(\omega)||S_2(\omega)|} , \quad (4)$$

where S_1 and S_2 are the Fourier transforms of s_1, s_2 and ω is the angular frequency $2\pi f$. The cross-spectrum is a function of the spacing between the hydrophones and the directionality and spatial coherence of the noise field. A non-zero imaginary part of the cross-spectrum indicates an anisotropic component in the noise field. The imaginary part is also proportional to the intensity of the propagating component of the noise field.

For a spatially isotropic noise field, the real part of the normalized cross-spectrum has the simple analytical form

$$S_{1,2}(kd) = \frac{\sin(kd)}{kd} \quad (5)$$

where $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength and d is the spacing between the hydrophones (Ref. 5). Equation 5 is plotted in Fig. 2 along with the cross-spectrum of two simulated isotropic noise time series.

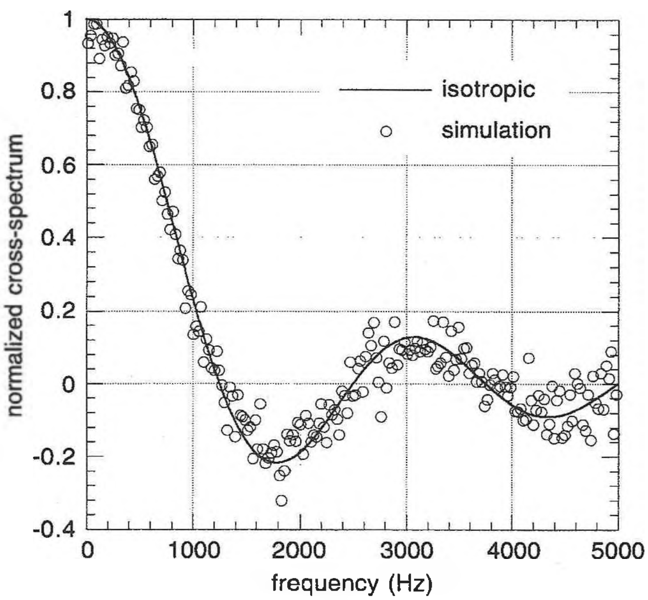

Fig. 2 Real part of cross-spectrum $S_{1,2}(\omega)$ for isotropic noise field and cross spectrum of two simulated time series.

To generate a pair of noise time series with a pre-defined cross-spectrum $S_{1,2}$, two Gaussian time series and their Fourier transforms were created. The phase differences $\Delta\phi$ between the complex Fourier components of S_1 and S_2 at each frequency interval were then found. The phase of the Fourier transforms were then shifted so that $\Delta\phi = \cos^{-1}[\mathrm{Re}(S_{1,2})]$. Thus the relative phases of the frequency components are adjusted so that the real part of the resultant cross-spectrum is equal to $S_{1,2}$ without affecting the magnitude of the Fourier coefficients, or equivalently, the power spectrum of the time series. The imaginary part of the cross-spectrum is a function of the phase shift and is given by $\mathrm{Im}(S_{1,2}) = \sin^{-1}(\Delta\phi)$. For isotropic noise, the time average of the imaginary part of $S_{1,2}$ goes to zero. In our simulation $\mathrm{Im}(S_{1,2})$ is constrained to average out to zero by alternating the signs of the phase shifts applied to S_1 and S_2 on successive iterations. The final step is to reverse Fourier transform S_1 and S_2 yielding two time series with the same individual statistics as $s_1(t)$, $s_2(t)$ but with the desired complex cross-spectrum.

An example of a cross-spectrum of two simulated isotropic noise time series is shown in Fig. 2 along with the theoretical cross-spectrum given by Eq. 5. The imaginary part of the calculated cross-spectrum (not shown) had a mean of 0.001 with a standard deviation of 0.071. The parameters of the simulation were d = 0.6 m and a sampling frequency of 10 kHz. The cross-spectrum shown in Fig. 2 is the average of results from 100 blocks of 4096 time series samples which is equivalent to a total of 41 s of time series data.

The above approach to simulating correlated noise is computationally efficient and can produce time series pairs with an arbitrary real cross-spectrum but the imaginary part of the cross spectrum must be zero. This is sufficient to represent any noise field which is symmetrical about the axis of the hydrophones. Noise with an arbitrary power spectrum can be generated by multiplying S_1 and S_2 by the appropriate weighting function.

3. Monte Carlo method

The noise simulation method described in Sect. 2 is limited to generating noise where the imaginary part of the cross spectrum is zero. However, noise fields are often anisotropic and have non-zero $\mathrm{Im}(S_{1,2})$. To handle these cases a third simulation technique based on the Monte Carlo method was developed.

The Monte Carlo algorithm randomly assigns directions and phases to a uniform distribution of monochromatic sources about a linear array of hydrophones. The resulting plane waves are weighted by an angular distribution of noise power, and then summed in the frequency domain as a "random walk" of phasors. By comparing the resulting phasor sums, the normalized cross-correlation coefficient is obtained. The complex cross-correlation function is then found by repeating the above process while stepping through frequency.

The angular weighting function applied to the noise sources depends on the physical situation being modelled, such as noise from an infinite plane surface above an absorbing bottom. The axis of the array can have an arbitrary orientation allowing anisotropic noise to be modelled for vertical or horizontal arrays. Time series possessing the appropriate cross-correlation statistics for a given angular weighting function can be generated for each hydrophone by distributing the phasor sums over frequency and then taking the inverse Fourier transform. The angular weighting function makes a Fourier transform pair with the desired cross-correlation spectrum, and thus either a cross-correlation or angular weighting function can be adapted as input for the generation of suitable time series.

4. References

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