Source Localization using Regularized Matched-mode Processing

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BACKGROUND

Localizing an acoustic source in the ocean is an important problem in underwater acoustics [1]. Matched-field processing (MFP) methods localize a source by matching acoustic pressure fields measured at an array of sensors with modelled replica fields computed for a grid of possible source locations. Matched-mode processing (MMP) consists of first decomposing the measured fields into their constituent modal components, and then matching the resulting mode excitations with modelled replica excitations. An advantage of MMP over MFP is that subsets of the complete mode set can be considered (e.g., in cases where seabed geoacoustic properties are poorly known, the matching can be applied only to low-order modes which interact minimally with the bottom). A disadvantage of MMP involves the modal decomposition itself. Modal decomposition represents a linear inverse problem that is non-unique and can be unstable (small errors on the data can lead to large errors on the solution). In particular, standard modal decomposition methods used in MMP can give poor solutions when the inversion is ill-posed due to an inadequate sampling of the acoustic field [1]. This paper develops a new approach to modal decomposition and MMP, referred to as regularized matched-mode processing (RMMP) [2].

THEORY

The normal-mode model for the acoustic pressure field p at a range r and depth z is given by

$$p(r,z) = \frac{e^{i\pi/4}\sqrt{2\pi}}{\rho(z_s)} \sum_{m=1}^{M} \phi_m(z)\phi_m(z_s)\frac{e^{ik_m r}}{\sqrt{k_m r}}, \qquad (1)$$

where ϕ_m and k_m represents the mth mode function and wavenumber, respectively, and M is the total number of propagating modes. The field recorded at a vertical line array (VLA) of N sensors can be written

$$\mathbf{A}\,\mathbf{x} = \mathbf{p}, \qquad (2)$$

where $\mathbf{p}$ is a vector of the measured acoustic pressures, $\mathbf{A}$ is an $N \times M$ matrix with columns consisting of the sampled mode functions, and $\mathbf{x}$ is a vector of the received mode excitations

$$\mathbf{x} = \left[\ \phi_1(z_s)\frac{\exp[ik_1 r]}{\sqrt{k_1 r}}, \ldots, \phi_M(z_s)\frac{\exp[ik_M r]}{\sqrt{k_M r}}\ \right]^T. \qquad (3)$$

Modal decomposition consists of solving (2) for an estimate $\bar{\mathbf{x}}$ of the true mode excitations $\mathbf{x}$.

The matrix $\mathbf{A}$ is orthogonal if the orthonormal mode functions $\phi_m(z)$ are adequately sampled over their entire extent. However, such sampling is often not possible. If the array contains fewer sensors than modes, the higher-order modes will be spatially aliased, and the inversion is singular. If the array spans too small a fraction of the water column, the lower-order modes will be poorly sampled, leading to an ill-posed inversion. Typically, singular value decomposition (SVD) or zeroth-order regularization is applied to stabilize the inversion; however, it should be noted these methods are based on determining the "smallest" solution in the sense that $|\bar{\mathbf{x}}|$ is as close to zero as possible. These smallest-model approaches produce a mathematical solution to the inverse problem (i.e., a stable solution that fits the data), but do not necessarily produce the most physically-meaningful solution.

A more general approach is regularized inversion, which minimizes an objective function Ψ that combines a term representing the data mismatch and a regularizing term incorporating an arbitrary *a priori* estimate $\hat{\mathbf{x}}$

$$\Psi = |\mathbf{G}(\mathbf{A}\,\bar{\mathbf{x}} - \mathbf{p})|^2 + \theta\,|\mathbf{H}(\bar{\mathbf{x}} - \hat{\mathbf{x}})|^2. \qquad (4)$$

In (4), $\mathbf{G}$ is a diagonal matrix with the reciprocals of the estimated data standard deviations on the main diagonal, $\mathbf{H}$ is an arbitrary weighting matrix for the regularization, and θ is a trade-off parameter that controls the relative importance of the two terms in the minimization. Minimizing Ψ with respect to $\bar{\mathbf{x}}$ leads to

$$\bar{\mathbf{x}} = \hat{\mathbf{x}} + [\mathbf{A}^\dagger\mathbf{G}^\dagger\mathbf{G}\,\mathbf{A} + \theta\,\mathbf{H}^\dagger\mathbf{H}]^{-1}\mathbf{A}^\dagger\mathbf{G}^\dagger\mathbf{G}\,(\mathbf{p} - \mathbf{A}\,\hat{\mathbf{x}}). \qquad (5)$$

An appropriate choice for θ is the value that produces a χ^2 data misfit (first term, right side of eq. 4) equal to $2N$, the expected value for N complex data. Although a closed-form solution for θ does not exist, it can be determined efficiently using Newton's method with analytic partial derivatives [2].

Applying a trivial prior estimate $\hat{\mathbf{x}} = \mathbf{0}$ in (4) and (5) leads to the standard zeroth-order regularized solution. RMMP is based on the conjecture that incorporating a physically-meaningful prior estimate can produce a better solution than standard MMP inversion techniques. The underlying idea makes use of the modelled replica mode excitations calculated for the search grid. The replica excitations for each grid point are computed via a normal-mode forward model, which provides a stable, noise-free solution. Thus, if the source is actually located at a particular grid point, the replica excitations computed for that point provide an ideal *a priori* estimate for the modal decomposition. This observation provides the basis for RMMP: an independent regularized inversion of the acoustic fields for the mode excitations is carried out prior to matching with the replica excitations for each grid point, using the replica itself as the prior estimate $\hat{\mathbf{x}}$. For grid points at or near the actual source location, using the corresponding replica as an *a priori* estimate and minimizing $|\bar{\mathbf{x}}-\hat{\mathbf{x}}|$ should provide a more meaningful solution than minimizing $|\bar{\mathbf{x}}|$. For grid points away from the source location, this procedure provides a stable inversion, although the regularization may not be particularly physical. At every grid point, RMMP produces the maximum match possible (subject to the data) between the replica and the constructed modal excitations.

EXAMPLE

This section compares source localization results obtained using RMMP, MMP and MFP for a realistic synthetic testcase. The ocean environment consists of a 300-m water column with a typical N.E. Pacific sound-speed profile overlying a 50-m thick sediment layer and semi-infinite basement. The sediment layer has a compressional speed of $c_p = 1650$ m/s, shear speed of $c_s = 300$ m/s, density of ρ=1.6 g/cm^3. The basement has $c_p = 2000$ m/s, $c_s = 800$ m/s and $\rho = 2.1$ g/cm^3. The acoustic source is located at (r, z)=(6 km, 100 m). This environment supports 12 propagating modes at the source frequency of 40 Hz, as shown in Fig. 1. The synthetic data for the testcase were computed using the ocean environment described above; however, all replicas used in inversion were computed for an environment with the basement compressional speed in error by 300 m/s (i.e., using 2300 m/s instead of 2000 m/s). The mismatched environment supports 14 modes (Fig. 1).

Source localization results are considered for signal-to-noise levels SNR = 15, 5, 0 dB, and a variety of different VLA configurations. To compare the different localization approaches for noisy data, 100 different random realizations of spatially-uncorrelated Gaussian noise were added to acoustic field data computed using a wavenumber integration model. The search grid extended from 0–12 km in range with a range increment of 100 m,

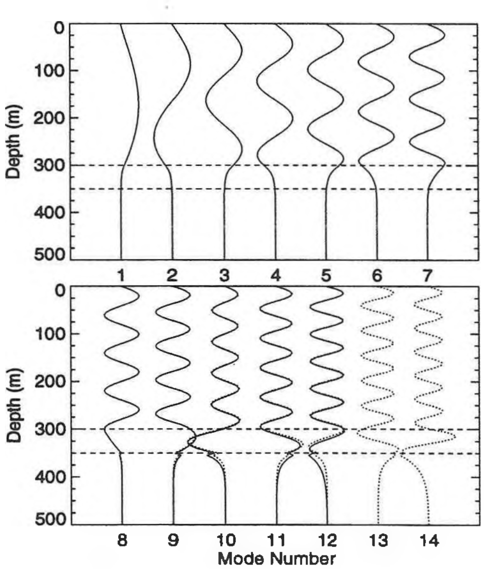

Fig. 1 Normal modes supported by the known and mismatched ocean environments shown by solid and dotted curves, respectively. Dashed lines denote the water-sediment and sediment-basement interfaces.

and 0–300 m in depth with a depth increment of 10 m. The estimated source location for each realization of noisy data corresponds to the grid point at which the match between the measured and replica fields (MFP) or mode excitations (RMMP and MMP) was a maximum, employing the standard Bartlett correlator. The performance of the various methods is quantified by the probability of correct localization P, taken to be the fraction of times that the localization is within $\pm$ 200 m in range and $\pm$ 10 m in depth about the true source location. For the matched-mode methods, only the eight lowest-order modes were included in the matching process to reduce the effects of environmental mismatch. For RMMP, H concentrated the regularization on the modes retained. MMP employed a zeroth-order regularization.

The results of this study are shown in Fig. 2 for VLAs consisting of from 6 to 12 sensors spanning the water column (Fig. 2a-c), and for VLAs consisting of 12 sensors with apertures spanning various fractions of the water column from 0.5 to 1 (Fig. 2d-e). Figure 2 shows that RMMP produced substantially better localization results than MMP or MFP for all array configurations, particularly for low to moderate SNRs.

REFERENCES

[1] A. Tolstoy, 1993. *Matched-field processing for underwater acoustics*, World Scientific, Singapore.

[2] N. E. Collison, 1999. Regularized matched-mode processing for ocean acoustic source localization, *M.Sc. thesis*, University of Victoria, Victoria BC, 90 p.

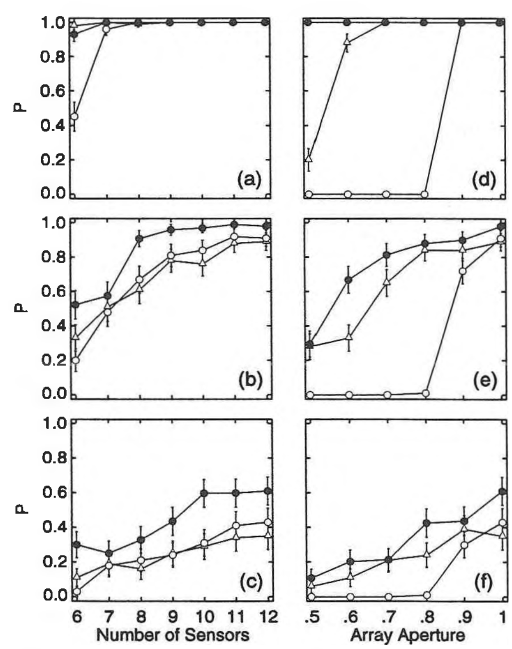

Fig. 2 Probability of correct localization P for MFP (triangles), MMP (open circles), and RMMP (filled circles). Results are given for the under-sampled cases for SNRs of (a) 15 dB, (b) 5 dB, and (c) 0 dB. (d)–(f) are the same as (a)–(c), but give results for short-aperture cases. Error bars denote 90% confidence intervals.