From both experimental and theoretical investigations, it is evident that soft marine sediments such as mud, clay, and silt have profiles of shear wave speed vs. depth that can be described by a "power-law" relation of the form

\[ c_s(z) = c_0 z^v, \]

in which \( c_s \) is the shear speed, \( z \) is the depth below the seafloor, and \( c_0 \) and \( v \) are constant parameters. The parameter \( c_0 \) represents the shear speed at unit depth, while \( v \) is a dimensionless parameter in the range 0\( < v < 1 \). We have determined that seismo-acoustic wave propagation in such profiles exhibits some peculiar features that depend critically on the value of \( v \).

Power-law profiles are mathematically singular, in the sense that the shear speed goes to zero with infinite slope at zero depth. Conventionally, one models seismo-acoustic wave propagation in a stratified elastic medium by dividing the layer into a stack of homogeneous sub-layers. In this way, physical effects associated with gradients of material properties are approximated by discrete coupling terms at the boundaries between the sub-layers. With an appropriate layering scheme, one expects results obtained with the stacked-layer model to converge to the presumed results of the continuous model as the number of layers is increased. However, for the power-law profile such convergence is glacially slow, owing to the very large gradients near the singularity. We have developed analytical techniques to model seismo-acoustic phenomena in power-law shear speed profiles, avoiding the pitfalls of stacked-layer models, provided that the shear speed is small in comparison with the compressional speed. Mathematically, this corresponds to "low-speed" elastic waves in the limit of infinite compressional speed. One could call these materials "incompressible solids". Some surprisingly simple results emerge, which we can only sketch in this summary paper.

Chapman (1997) and Godin and Chapman (1999) have shown that an ocean bottom seismometer placed on the seabed over a layer of soft sediment with a high-impedance basement will show very large resonances in horizontal displacement. These resonances are excited by the infrasonic ambient noise field in the ocean, the energy having been converted from compressional waves to shear waves at the sediment/basement interface. Naturally, the spacing of the resonances is governed by a reference frequency that is the inverse of the two-way travel time of shear waves within the sediment; however, the precise location of the fundamental frequency is strongly dependent on the value of the parameter \( v \), as indicated in Figure 1.

In the same medium, there may exist seismo-acoustic interface waves at the water/sediment boundary. Osier and Chapman (1996) observed these at the same site that displayed the ambient noise resonances, using the same instrument. They modelled the dispersion of these waves (i.e. curves of group speed vs. frequency) using a stacked-layer model, and the resulting shear speed staircase implied a continuous power-law profile with \( c_0 = 23 \) and \( v = 0.61 \). The new analytic approach allows inversion of these two profile parameters directly from the dispersion data. Moreover, the analytic approach provides the elegant result that both the phase speed and the group speed of interface waves in power-law profiles scale with frequency according to \( f^{\nu(v-1)} \). This scaling law is observed not only in our own data but in data of other researchers, as shown in Figure 2. The full details of this last result are being prepared for journal publication.


