NUMERICAL MODELING OF POROUS-ELASTIC MATERIALS USING HIERACHICAL ELEMENTS

S. Rigobert[1], N. Atalla[2] and F.Sgard[1]

1) Laboratoie des sciences de l'habitat, DGCB URA CNRS 1652, ENTPE, rue Maurice Audin, 69518 Vaulx en Velin cedex, France

2) G.A.U.S, Université de Sherbrooke, Sherbrooke, Québec, J1K 2R1, Canada

INTRODUCTION

Porous materials are increasingly used in many industries such as aeronautics, automobile, building acoustics to improve the reverberent properties of rooms and increase the transmission loss of multilayered structures. Recently, FE models based on either {u,U}[1] or {u,P}[2] formulation have been extensively used to model such structures. These models, based on classical linear finite elements, require refined meshings to insure convergence, and consequently a great number of degrees of freedom.

The present approach aims at reducing the number of degrees of freedom while keeping the accuracy of the results. To achieve that goal, a {u,P} formulation based on hierarchical elements (high interpolation order shape functions) is implemented. The results for a single porous material bonded onto a rigid wall are presented in the case of acoustical and mechanical excitations, and different boundary conditions on the lateral faces. The performance of the approach is underlined through a comparison with classical poroelastic linear elements. The effect of different interpolation orders for the solid and the fluid phase is discussed in the oral presentation.

THEORY

In the following, the case of a single porous material is considered. The weak {u,P} formulation is given by:

$$\underbrace{\int_{\Omega p} \underline{\underline{\widetilde{\sigma}}}^S(\underline{u}^S) : \underline{\underline{\epsilon}}^S(\underline{u}^S) d\Omega - \widetilde{\rho}\omega^2 \int_{\Omega el} \underline{u}^S \cdot \delta \underline{u}^S d\Omega}_{\text{solid phase}} + \underbrace{\int_{\Omega p} \Big[\frac{h^2}{\omega^2 \widetilde{\rho}_{22}} \nabla p \cdot \nabla \delta p - \frac{h^2}{\widetilde{R}} p \delta p\Big] d\Omega}_{\text{fluid phase}}$$

$$\underbrace{- \int_{\Omega p} \Big[\gamma + h\Big(1 + \frac{\widetilde{Q}}{\widetilde{R}}\Big)\Big]\Big[\nabla p \cdot \delta \underline{u}^S + \underline{u}^S \cdot \nabla \delta p\Big] d\Omega - h\Big(1 + \frac{\widetilde{Q}}{\widetilde{R}}\Big) \int_{\Omega p} \Big[p \cdot div(\delta \underline{u}^S) + div(\underline{u}^S) \cdot \delta p\Big] d\Omega}_{\text{symetrical volume coupling terms}}$$

$$\underbrace{- \int_{\partial \Omega p} \Big[\underline{\underline{\sigma}}^t \cdot \underline{n}\Big] \cdot \delta \underline{u}^S dS - \int_{\partial \Omega p} h\big(U_n - u_n\big) \delta p dS}_{\text{natural boundary terms}} = 0 \qquad (1)$$

In equation (1), Ωp and $\partial \Omega p$ represent the poroelastic domain and its boundary, $\underline{u}^p$ is the solid phase displacement, p is the fluid pressure in the pores, $\underline{\underline{\widetilde{\sigma}}}^S$ and $\underline{\underline{\epsilon}}^S$ are the strain and stress tensors of the solid phase *in vacuo*, $\widetilde{\rho}$ and $\widetilde{\rho}_{22}$ are the complex dynamic solid and fluid phase mass density, h porosity, γ the coupling coefficient between the two phases, $\widetilde{Q}$ and $\widetilde{R}$ are poroelastic coeficients. Equation (1) is slightly different from the original form[2]. It allows for an easier application of the boundary conditions and coupling conditions through the natural boundary terms.

The theoretical foundation of hierarchical elements lies in the way the pressure in the pores and the displacement of the solid phase are interpolated at any point of the subdomain. On one 8-nodes brick element, the pressure in the pores is given by:

$$P(\xi, \eta, \zeta) = \sum_j \mathcal{N}_j q_j^{ph} + \sum_k \mathcal{G}_k q_k^{gen} \qquad (2)$$

Where ξ, η, ζ are the coordinates on the parent element and vary from -1 to 1. In equation (2), $\mathcal{N}_j$ and $\mathcal{G}_k$ are polynomial shape functions defined on the parent element. The associated variables q_j^{ph} and q_k^{gen} stand for the amplitude of these functions. Amplitudes q_j^{ph} $(1 < j < 8)$ are the amplitudes of the pressure in the pores at the element nodes, and $\mathcal{N}_j$ are the classical shape functions used in finite elements. The values of $\mathcal{N}_j$ are 1 at node j and 0 at any other node of the considered element. Amplitudes q_k^{ph} have no simple physical meaning, and are not used in classical finite element. The associated generalized functions $\mathcal{G}_k$ are divided in several categories: side modes, face modes, and internal modes. These functions are selected to make complete polynomials of ascending order p. Namely, the basis functions of the hierarchical variables are constructed using Legendre polynomials. Their number depend on the interpolation order p. For further details on the selection process, the reader is invited to refer to Babuška[3]. The approximation of the solid phase displacement on one element is achieved in the same way as for the pressure in the pores.

In the results section, two indicators are considered. For the solid phase, the mean square velocity in the three directions is computed. The mean quadratic velocity along the i direction is practically computed using the formula:

$$< v_i^2 >= \frac{\omega^2}{2\Omega_p} < u_n^{ph} \vdots u_n^{gen} >^* [M_i^S] \begin{Bmatrix} u_n^{ph} \\ \cdots \\ u_n^{gen} \end{Bmatrix} \qquad (3)$$

Where $< u_n^{ph} \vdots u_n^{gen} >$ is the row vector of the physical and generalized amplitudes relative to the solid phase displacement along direction i. $[M_i^S]$ is the mass matrix for the solid phase divided by $\widetilde{\rho}$. It contains coefficients corresponding to the direction of the displacement. In equation (3), (*) means

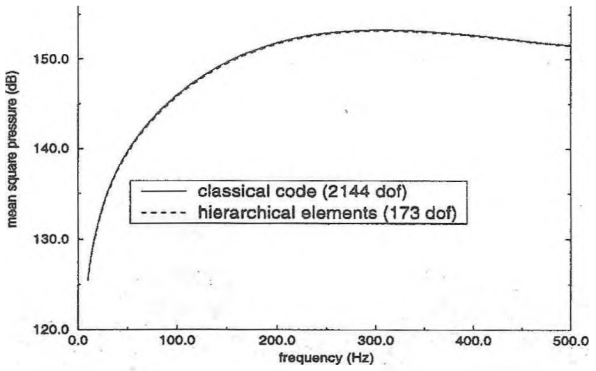

Figure 1: Single porous material on rigid wall with free edges, excited by piston motion

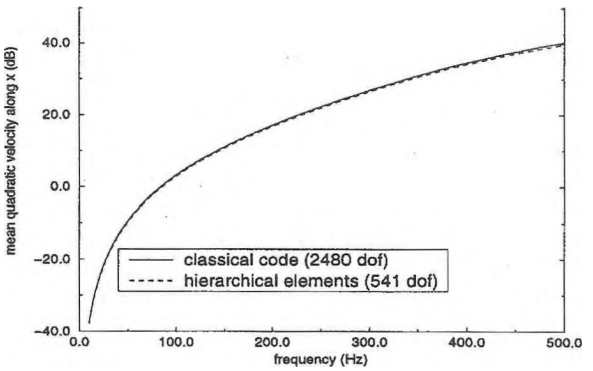

Figure 2: Single porous material on rigid wall with bonded edges, excited by incoming plane wave under normal incidence

Material	h	σ (kN.s/m^4)	α_∞	Λ (μm)	Λ' (μm)	ρ_S (kg/m^3)	N (kPa)	ν	η_S
FM2	0.90	25	7.8	226	226	300	286	0.4	0.265

Table 1: Properties of the material

complex conjugate.
For the fluid phase, the mean quadratic pressure is computed using the folowing formula:

$$< P^2 >= \frac{1}{2\Omega_p} < q_n^{ph} . q_n^{gen} >^* [M^f] \left\{ \begin{array}{c} q_n^{ph} \\ \cdots \\ q_n^{gen} \end{array} \right\} \qquad (4)$$

Where $< q_n^{ph} . q_n^{gen} >$ is the row vector of the physical and generalized amplitudes relative to the pressure in the pores. $[M^f]$ is the mass matrix divided by $h^2/\tilde{R}$.

RESULTS

The case of a single porous material of dimensions 0.35m*0.22m*0.05cm bonded onto a rigid wall is studied here. The properties of the material are given in Table 1. Two configurations are considered. First, the porous material is submitted to a piston motion and its lateral edges are free. Then, the material is excited by an incoming plane wave under normal incidence and the lateral edges of the porous material are bonded. The computed indicators, either the mean square velocity or the mean square pressure are compared to the results given by a FE code developped at the GAUS. This latter code based on classical poroelastic linear elements is chosen as reference and is insured to converge for the meshing considered. The results are presented on Figure 1 and 2. The number of degrees of freedom has been chosen so as to insure the convergence of all the indicators within 0.5dB from the results given by the classical code. For both cases, hierarchical elements enable to get very accurate results using only a reduced number of degrees of freedom. Actually, 2144 dof are needed for the first configuration and 2480 for the second configuration when the classical code is used. In comparison, the present approach requires 173 dof for the first configuration and 541 dof for the second configuration. The indicator chosen for the latter, namely the mean quadratic velocity along x direction is the one that has the most difficulties to converge.

CONCLUSION

In this paper, a FE code based on the $\{u, P\}$ formulation for a porous material and using hierarchical elements has been designed. The response for a single porous material has been computed and compared to the results given by classical elements. Hierarchical elements prove to give accurate results with less degrees of freedom.

REFERENCES

(1) Panneton, R. and Atalla, N. "Numerical prediction of sound transmission through finite multilaye systems with poroelastics materials." J.Acoust.Soc.Am., 100(1), **1996**
(2) Atalla, N. Panneton, R. and Debergues, P. "A mixed displacement pressure formulation for poroelastic materials" J.Acoust.Soc.Am., 104(3), **1998**
(3) Szabò, B. Babuška, I. "Finite element analysis", John Wiley and Sons, New York