NON-LINEAR ANALYSIS OFELECTROSTRICTIVE MATERIALS BY THE FINITE ELEMENT METHOD

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1. INTRODUCTION

New electrostrictive lead magnesium niobate ceramics (PMN) are promising materials for realizing actuators or high power transducers for macrosonics or underwater acoustics. Because of their large dielectric permittivity, PMN materials have strains roughly an order of magnitude larger than those of lead titanate zirconate (PZT) ceramics. However, the use of PMN as active material in actuators or transducers presents some difficulties : highly non-linear properties (Fig.1), temperature and frequency dependence of dielectric permittivity, DC bias field needed. To help in the design of PMN-based transducers, a numerical modeling capability is needed.

2. CONSTITUTIVE EQUATIONS OF PMN ELECTROSTRICTIVE CERAMICS

PMN electrostrictive materials are relatively new and complicated in behavior [1]. Non-linear constitutive models for electrostrictors are not as mature as models for piezoelectrics [2]. The model used in this paper is Hom's model [2,3]. Choosing the electric displacement and the stress as the independent state variables, the constitutive equations can be written at constant temperature:

$$S_{ij} = S^{D}_{ijkl} T_{kl} + Q_{ijmn} D_m D_n$$

$$E_m = -2Q_{ijmn} D_n T_{ij} + \frac{\delta_{mn}}{k|D|} \operatorname{atanh}\left(\frac{|D|}{P_s}\right) D_n$$

where S_{ijkl} is the elastic compliance at constant electric displacement, T_{kl} is the stress, D_m is the electric displacement, E_m is the electric field, S_{ij} is the strain, Q_{ijmn} is the electrostrictive coefficient, δ_{mn} is the Kronecker symbol, P_s is the spontaneous polarization and k is a new material constant.

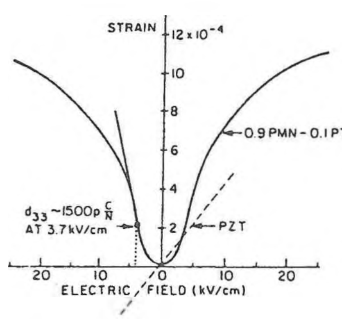

Fig. 1. Typical strain-electric field for electrostrictive and piezoelectric ceramics [4]

3. FINITE ELEMENT FORMULATION

The electrostrictive finite element is developped in the ATILA code [5,6]. Starting with Newton's law, Gauss's law and the equation of electrostriction in the electrostrictive material, Helmholtz's equation in the fluid and the Sommerfeld's radiation condition, the method of weighted residuals is used for the static and transient analyses to get the finite element formulation [7]. For transient analysis, the set of equations for the electrostrictive structure in a fluid domain is written:

$$\begin{bmatrix} [M] & [0] & [0] \\ [0] & [0] & [0] \\ \rho_f^2 c_f^2 [L]^T & [0] & [M_1] \end{bmatrix} \begin{Bmatrix} \ddot{\underset{\sim}{U}} \\ \ddot{\underset{\sim}{\phi}} \\ \ddot{\underset{\sim}{P}} \end{Bmatrix} + \begin{bmatrix} [0] & [0] & [0] \\ [0] & [0] & [0] \\ [0] & [0] & [D_1] \end{bmatrix} \begin{Bmatrix} \dot{\underset{\sim}{U}} \\ \dot{\underset{\sim}{\phi}} \\ \dot{\underset{\sim}{P}} \end{Bmatrix}$$

$$\begin{bmatrix} [K_{UU}] & [K_{Uf}] & [L_U] \\ 2[K_{Uf}]^T & [K_{UU}] & [0] \\ [0] & [0] & [H]+[D_0] \end{bmatrix} \begin{Bmatrix} \underset{\sim}{U} \\ \underset{\sim}{\phi} \\ \underset{\sim}{P} \end{Bmatrix} = \begin{Bmatrix} \underset{\sim}{F} \\ -\underset{\sim}{Q} \\ \underset{\sim}{0} \end{Bmatrix}$$

where $\underline{U}$, $\underline{\phi}$, $\underline{P}$, $\underline{F}$ and $\underline{Q}$ are the vectors of the nodal values of the displacement, the electric potential, the pressure, the external force and the electric charge respectively. $[K_{uu}]$, $[K_{up}]$, $[K_{pp}]$ and [M] are the classical stiffness, piezoelectric, dielectric and consistent mass matrices of a piezoelectric finite element model. [0] is the zero matrix. For the fluid:

$$[D_0] = \frac{\rho_f c_f^2}{R} [D]$$

$$[D_1] = \rho_f c_f [D]$$

where [H], $[M_1]$, [D] are the stiffness, consistent mass and damping matrices, ρ_f and c_f are the density and the velocity of the fluid and R is the radius of the spherical boundary which limits the fluid mesh.

4. VALIDATION

4.1. STATIC ANALYSIS OF A PMN BAR

To validate the previous development, a long electrostrictive bar with electrodes located at both ends is analyzed at ambient temperature. A static mechanical force is applied at both ends and a quasi-static (1 Hz) electric field parallel to the length is prescribed. Numerical results are compared to measurements made on a PMN-PT-La (0.90/0.10/1%) bar at NUWC New London [8]. The finite element mesh of the bar consists of two axisymmetrical electrostrictive elements. These elements are eight-noded isoparametric quadrilaterals.

Figure 2 presents the quasi-static strain versus quasi-static applied electric field for various prestresses. The static strain is not measured. In both figures, good agreement is obtained between computed results and measurements in a broad range of applied electric fields and prestresses.

4.2. DYNAMIC ANALYSIS OF A PMN BAR

We study the dynamic response of the same PMN bar. Two types of excitations are considered:

• a step in voltage which generates a vibration of the bar at constant

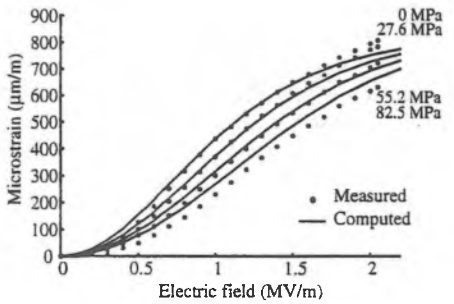

Fig. 2. Strain versus electric field at various prestresses

electric field E. The corresponding natural frequency is noted f_E.

• a step in charge which generates a vibration of the bar at constant electric displacement D. The corresponding natural frequency is noted f_D.

Knowing these two frequencies, the coupling coefficient of the bar is calculated from the expression:

$$k^2 = \frac{f_D^2 - f_E^2}{f_D^2}$$

This expression is similar to Ikeda's coupling coefficient definition from elastic constant c^D and c^E [9].

Figure 3 displays the displacement at the end of the bar versus time for an initial voltage of 2000 volts. The mesh of the bar is unchanged. The thin line represents the vibration at constant E and the bold line the vibration at constant D. The corresponding frequencies are obtained using Discrete Fourier Transform. In figure 4, the coupling coefficient is represented for various initial voltages and voltage steps. Saturation is observed around 45% at high electric field. The observed electrostrictive coupling coefficients are smaller than usual piezoelectric coupling coefficient of PZT8 ceramics (k_{33} around 60%).

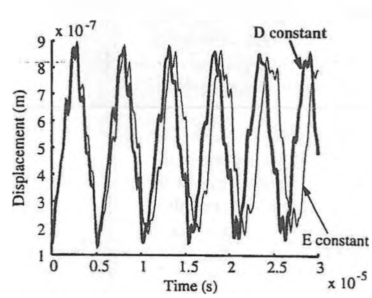

Fig. 3. Displacement at the end of a PMN bar submitted to a voltage (2000 V) or charge (23.8 nC) step versus time (initial voltage of 2000 V)

4.3. Transient radiation of an electrostrictive PMN sphere

In this section, we study the response of a PMN spherical shell to a step in voltage. The finite element mesh consists of 6 axisymmetrical electrostrictive (eight node quadrilateral) elements. Figure 5 displays the displacement in the middle of the PMN shell versus time for LC (Lumped Constant model) and FE (Finite Element) models for an initial voltage of 4000 V and a voltage step of 1000 V. The thick line correspond to the analytical model, the thin line correspond to the finite element model. A very good agreement is observed between the LC and FE models. Figure 6 displays the deformation of the sphere (full line) versus the structure at rest (dashed line) at time t = 8 10-5s. We notice that the PMN shell is always in compression.

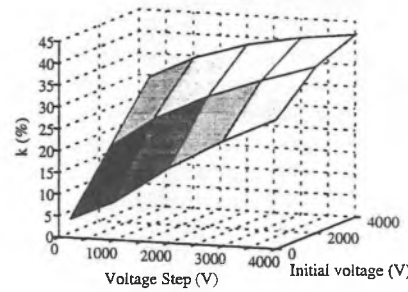

Fig. 4. coupling coefficients of a PMN bar for various initial voltages and steps

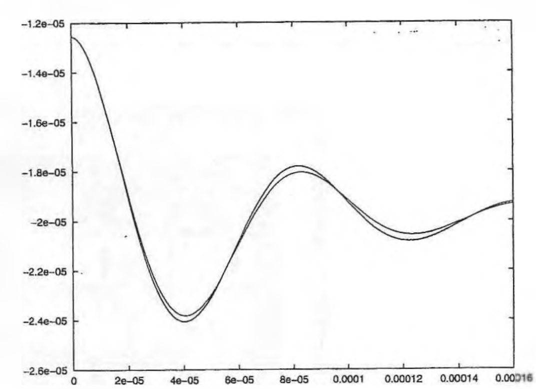

Fig. 5. Displacement of the PMN shell submitted to 1000 V step (initial voltage of 4000 V) versus time. Thick line: semi-analytical model (LC), thin line: finite element model (FE)

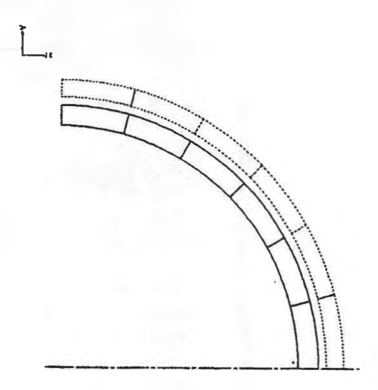

Fig. 6. Strained structure of the PMN spherical shell submitted to an initial voltage and a voltage step. Full line: strained structure, dashed line: initial structure

An Efficient Method Based On Multipole Expansion For Predicting The Sound Power Of Baffled Plane Plates

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1. INTRODUCTION

To evaluate the radiated power from a vibrating structure, the most popular approach consists in the integration of the active acoustic intensity normal to the structure surface. This method needs the evaluation of the pressure field on the surface of the structure. The latter can be calculated by classical discretization methods such as the boundary elements method (BEM). Nevertheless, this methodology suffers from the computational cost associated to forming and solving the frequency dependent linear system. If the system is large (complex structure or extended fre
uency range), the memory of the computer can be the limiting factor so that an out of core solver may be required which adds to the computational cost. To answer to this limitation, an iterative solver [2] can be used in order to avoid the construction of the discretized full matrix system in memory but the efficiency and stability of the algorithm can be disastrous.

To evaluate the acoustic power radiated by a collection of M sources (or a distributed volume source), the simple way consists in the integration of the far-field pressure over a sphere surrounding the M sources [6]. This operation is very expensive due to the need to calculate the field at numerous evaluation points on the sphere. Indeed, the operation cost is of order M at each evaluation point. Using a multipole expansion [4,7] for the set of source points, the field can be efficiently evaluated at points sufficiently far from a sphere enclosing the source points. This condition is always respected in the case of the evaluation of the radiated power since we need only to integrate the far-filed pressure over a sphere. A similar methodology has been already developed by Atalla and al [3]. However, the authors develop a multiple multipole expansion because they use only the first three terms for the expansion.

In the case of baffled plane plate, the pressure field is governed by the Rayleigh's integral [6]. Since this integral can be developed in a multipole expansion, it is shown in this paper that the radiated power can be found accurately and cheaply by integrating the far-field pressure over a hemisphere.

2. EQUATIONS

Consider a set of M punctual acoustic sources in a sphere S_a with centre $\mathbf{a}$. Their positions are $(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ and their intensities are given by $(q_1, \ldots, q_M)$. The acoustic pressure field caused by these sources at a point $\mathbf{r}$ can be written

$$p(\mathbf{r}) = \sum_{l=1}^{M} q_l \frac{\exp^{-ik|\mathbf{r}-\mathbf{x}_l|}}{4\pi\,|\mathbf{r}-\mathbf{x}_l|} = -\frac{ik}{4\pi}\sum_{l=1}^{M} q_l h_0(k\,|\mathbf{r}-\mathbf{x}_l|) \quad (1)$$

where

$$h_0(k\,|\mathbf{r}-\mathbf{x}_l|) = \frac{\exp^{-ik|\mathbf{r}-\mathbf{x}_l|}}{-ik\,|\mathbf{r}-\mathbf{x}_l|}$$

and k is the wave number. In this way, the cost of the evaluation at one point is of order M. If the evaluation point is outside the sphere S_a, the field can be expanded in an outer multipole expansion of the form

$$p(\mathbf{r}) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} C_n^m h_n(k\,|\mathbf{r}-\mathbf{a}|) Y_n^m\left(\frac{\mathbf{r}-\mathbf{a}}{|\mathbf{r}-\mathbf{a}|}\right) \quad (2)$$

with Y_n^m the spherical harmonics and h_n the spherical Bessel functions.

Using the Gegenbauer formula [1,4] which writes the spherical Bessel functions in terms of spherical harmonics, the multipole expansion coefficients, C_n^m, can be written after few algebraic manipulations,

$$C_n^m = \frac{-ik(2n+1)}{4\pi}\sum_{l=1}^{M} q_l j_n(k\,|\mathbf{x}_l-\mathbf{a}|) Y_n^{-m}\left(\frac{\mathbf{x}_l-\mathbf{a}}{|\mathbf{x}_l-\mathbf{a}|}\right) \quad (3)$$

The expansion (2) is exact for an infinite number of terms. Nevertheless, we always truncate the expansion to N so that each evaluation point has to be sufficiently far from the sphere S_a. Also, the terms $h_n(x)$ in the expansion (2) grow quickly with n for $n>x$. This fact can cause numerical instability. One more time, this difficulty vanishes if the point $\mathbf{r}$ is far from the centre of the expansion $\mathbf{a}$. From equation (3), we can see that coefficients C_n^m do not depend on the observer position. Thus, once these coefficients are computed for a set of source points, the far-field at a large number of observer points can be cheaply evaluated using equation (2). This is the basic idea behind the efficiency of the approach for evaluation of the radiated power.

The acoustic power radiated by the collection of acoustic source points can be evaluated by integrating the far-field pressure over a sphere [6]

$$\Pi_{ac} = \int_{S_R} \frac{|p|^2}{2\rho_f c} dS_R = \int_0^{2\pi}\int_0^{\pi} \frac{|p(R,\vartheta,\varphi)|^2}{2\rho_f c} R^2 \sin\vartheta\, d\vartheta\, d\varphi$$

Introducing the multipole expansion (2) in the last expression, we obtain

$$\Pi_{ac} = \frac{1}{2\rho_f c}\int_0^{2\pi}\int_0^{\pi} \left| R\sum_{n=0}^{N} h_n(kR) \sum_{m=-n}^{n} C_n^m Y_n^m(\vartheta,\varphi)\right|^2 \sin\vartheta\, d\vartheta\, d\varphi$$

Posing and , the last equation becomes

$$\Pi_{ac} = \frac{1}{2\rho_f c}\int_0^{2\pi}\int_{-1}^{1} \left| R\sum_{n=0}^{N} h_n(kR) \sum_{m=-n}^{n} C_n^m Y_n^m(\arccos(x),\varphi)\right|^2 dx\, d\varphi$$