A INDEX TO PREDICT THE STABILITY OF DECENTRALIZED ADAPTIVE FEEDBACK ACTIVE NOISE CONTROL SYSTEM

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1. Introduction

A large number of industrials applications like transformers or rotating machinery are concerned by radiation of unwanted periodic noise. Typically, centralized techniques are used to achieve the control of these systems. Nevertheless, the implementation of such a control requires a considerable processing power, rapidly growing with the number of actuators. One way to avoid this problem is to use a number of independently operating control systems in which, a subset of controllers drives a smaller number of secondary sources. However, such a control strategy does not take into account the different interactions between the actuators and error sensors, which possibly lead to instability and the degradation of the overall system performances.

This paper presents a decentralized adaptive feedback controller developed for the control of periodic disturbances. The study of the stability of the feedback loop and the the stability of the control is effectuated in the frequency domain. Simulations of the control are also presented to verify the accuracy of this analysis.

2. Description of the control system

In most of the practical applications, the only information available to implement a control is the knowledge of the physical plant of the system to be controlled h. In this case, the controller implemented for the control must have a feedback structure. Moreover, when the size of the system to be controlled is large, it is useful to achieve a decentralized control strategy, wich consists to employ independant operating controller. The equivalent controller of the system C (shown in figure 1) is thus a diagonal controller wich doesn't take into account all the interactions between the control units. If these interactions are too important to be neglected, the feedback loop will be unstable.

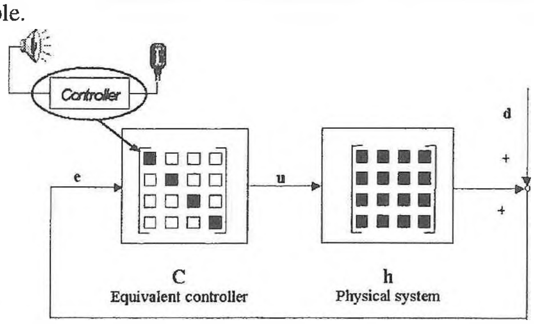

Figure 1: Representation of the decentralized control system.

The decentralized controller consider in this work is composed of M independant control unit. Each control unit, constituted of a loudspeaker and a error microphone, is a feedback loop with an internal model control architecture [1]. A filtered x-LMS algorithm is used to adapt the command signal of the loudspeaker to minimize the sound pressure at the microphone as represented in figure 2.

The LMS gradient descent algorithm implemented for the adaptation of each unit is defined in the frequency domain for the lth unit as:

$$\Gamma_l(k+1,\omega_0) = \Gamma_l(k,\omega_0) - 2\frac{\mu}{\left|\hat{d}_l(k,\omega_0)\right|^2}\hat{h}^*_{ll}(\omega_0)\ \hat{d}^*_l(k,\omega_0)\ e_l(k,\omega_0) \quad (1)$$

where ω_0 is the frequency of the primary disturbance, * denotes the complex conjugate and μ is step size parameter.

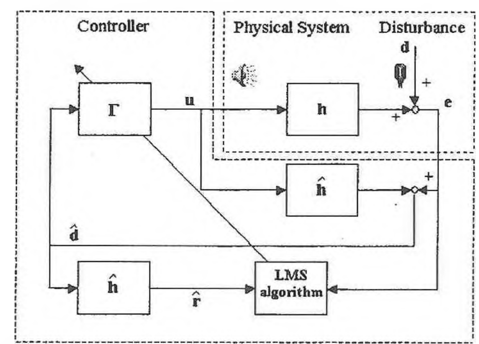

Figure 2: Bloc diagram of each control unit.

The study of the adaptation process realized in [2] has shown that, during the adaptation, each control filter converges in a quasi-straight line from its initial value (fixed to zero), to its steady state value $\Gamma_l^{opt}(\omega_0)$. This steady state value, which ensures an error signal equals to zero at the microphone is defined as:

$$\Gamma_l^{opt}(\omega_0) = \frac{-1}{\hat{h}_{ll}(\omega_0)} \quad (2)$$

Furthermore, the position of each control filter on its convergence path at any iteration k can be associated to a parameter β_k given by:

$$\beta_k = \frac{\Gamma_l(k,\omega_0)}{\Gamma_l^{opt}(\omega_0)} = 1 - e^{-2k\mu\left|\hat{h}_{ll}(\omega_0)\right|^2} \quad (3)$$

As the control performance depends on the value of the control filter, this parameter β_k is called performance index [2]. Indeed, the closer the control filter is of its steady state value, the better the attenuation measured at the microphone will be. If k=0 then $\beta_k = 0$, the control filter stays on its initial value and no control of the disturbance is accomplished. In the same way, if $k \to \infty$ then $\beta_\infty = 1$, the control filter has reached its optimal value and a perfect rejection of the perturbation is obtained with a large attenuation measured at the error microphone.

3. Stability of the feedback loop

The stability of the feedback loop of the decentralized control system presented in this section is realized by assuming the adaptation process "frozen" at an iteration k. The correponding values of the control filters at this iteration, are associated to a mean value of the index performance β calculated from equation 3. According to the classical Nyquist criterion, the closed loop system will be stable if and only if, the map of $L(s,\beta) = det\,(I + L_H(s)\,\overline{\boldsymbol{H}}(s,\beta))$ evaluated on the standard Nyquist D-contour doesn't encircle the origin point [2]:

$$N(0, L(s,\beta)) = 0 \quad (4)$$

With:

$$\overline{\boldsymbol{H}}(s,\beta) = -\hat{\boldsymbol{h}}(s)\ \boldsymbol{C}(s,\beta)\ \left[\boldsymbol{I} - \hat{\boldsymbol{h}}(s)\ \boldsymbol{C}(s,\beta)\right]^{-1} \quad (5)$$

$$\boldsymbol{L_H}(s) = [\ \boldsymbol{h}(s) - \hat{\boldsymbol{h}}(s)\ ]\ \hat{\boldsymbol{h}}^{-1}(s) \quad (6)$$

$$\boldsymbol{C}(s,\beta) = [\boldsymbol{I} + \boldsymbol{\Gamma}(\beta)\ \hat{\boldsymbol{h}}(s)]^{-1}\ \boldsymbol{\Gamma}(\beta) \quad (7)$$

$$\boldsymbol{\Gamma}(\beta) = diag\left\{\frac{-\beta}{\hat{h}_{ll}(\omega_0)}\right\} \quad (8)$$

3. Simulations of the control

Different results obtained from the simulation of the control in the time domain for a system composed of 7 coplanar control units at frequency 240 Hz are now presented. The geometrical configuration of the control units used for these simulations is illustrated in figure 3. The physical plant h, was obtained from the experimental values of the 128-order FIR filters that modelled the transfer function between each loudspeaker and each microphone.

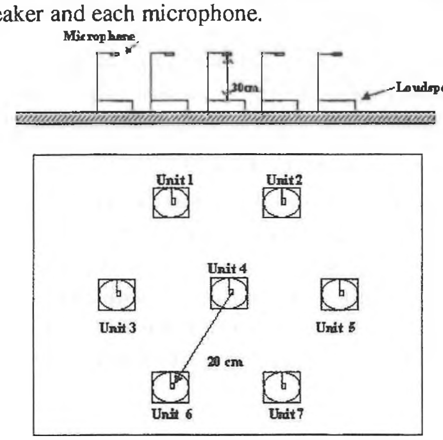

Figure 3: Configuration of the control simulation.

During these simulations, the adaptation process is frozen at different iterations k. The corresponding values of the control filters Γ (modelled by 32-order FIR filter) are then used to calculate the mean values of the performance index β and $L(s,\beta)$.

Figure 4 shows the evolution of the map $L(s,\beta)$ obtained for these different values of the index performance. According to this figure, the stability of the system is affected by the values of β with a decrease of its stability margin when β increases. The control system is find to be stable while the map of $L(s,\beta)$ doesn't encircle the origin which correspond to values of $\beta \leq 0.30$. For β=0.35, the map of $L(s,\beta)$ encircles the origin and the control system is unstable in regard of the Nyquist criterion.

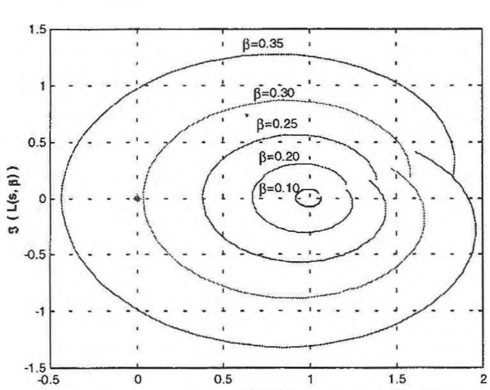

Figure 4: Map of L (s, β) on the Nyquist D-contour obtained for different values of β.

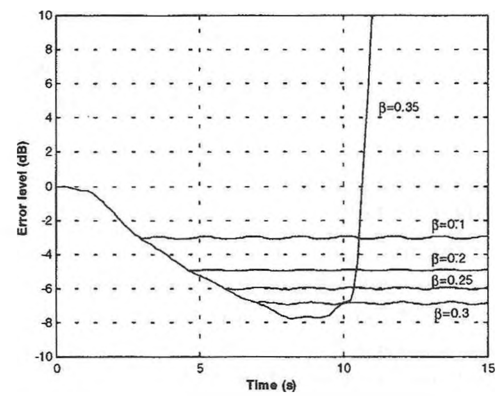

Figure 5: Sum of the temporal squared error signals measured at the microphones for different values of the index performance β.

The sum of the squared error signals at the microphones for the same values of β are represented in figure 5. As observed on this figure, if $\beta \leq 0.30$, the control filters progress on their convergence path while the adaptation process is actuated. The error signal at the microphones decreases for increasing values of β. When the adaptation is frozen, the control filters stop their convergence on a specific value and the error signal at the microphone cease its decrease. For β=0.35, a decrease of the error signal is also noticed at the begining of the adaptation until a large increase of the error signal is observed. This one corresponds to the instability of the control system expected by the Nyquist criterion. This can be explain by the fact that during the adaptation, the control filters reached some values which cause the instability of the feedback loop.

4. Conclusions

This paper presents a decentralized adaptive feedback control system of periodic noise. A condition, derived from the Nyquist criterion, is given in order to predict the stability of the control. It has be shown, that the positions of the control filters on their convergence path play an important role on the stability and the performance of the control system. The control of the parameter β, called performance index wich was associated to these positions, appears thus a good alternative to make a compromise between performance and stability. The manner to control β presented in this paper, was to stop the adaptation of the control filters on the desired values. However, this technique is difficult to implement in practical cases. Others procedures of control of this parameter could be developped and implemented like introducing a leaky coefficient in the LMS algorithm, or reinjecting a proportional part of the estimate signal of the pertubation in the calcul of this same estimate signal of the perturbation realized by the internal model.

References

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[2] E. Leboucher, P. Micheau, A. Berry and A. L'Espérance, "A decentralised adaptive feedback active noise control system of periodic sound in free space", Proceeding of Active 99, pp. 973-984, 1999.