FINITE ELEMENT MODELING OF DAMPING USING PIEZOELECTRIC MATERIALS

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1.0 INTRODUCTION

Damping of mechanical vibrations has attracted a great deal of interest because it can reduce noise and peak vibration amplitudes in systems. Piezoelectric materials possess properties which make them useful as dampers or control elements for structures. Piezoelectric materials have the following properties : they strain when an electrical field is applied accross and they produce a voltage under strain. In general, piezoelectric materials have the ability to transform mechanical energy into electrical energy and reciprocally. Hagood *et al* [1] have presented a passive damping mechanism for structural systems in which piezoelectric materials bonded to the structures are used. They present the possibility of dissipating mechanical energy with piezoelectric materials shunted with passive electrical circuits. More recently, Law *et al* [2] have studied the damping behavior of a piezoelectric material, shunted by a resistance, described by a mechanical energy conversion and dissipation by the material, instead of a change in stiffness.

The use of the finite element method to tackle these problems can strongly broaden the designer's possibility, particularly because it allows the modeling of any structure geometry, any materials as well as any external electrical circuit. The paper presents the finite element modeling of damping using piezoelectric materials, with the help of the ATILA code [3], that has been adapted to take into account an external electrical circuit. After the presentation of the formalism, two applications are presented.

2.0 FORMALISM

In order to be able to model any type of piezolectric transducer, without any restriction on the shape and the materials, a model has been developed, in the ATILA code, based on a variational principle. Classically, for an in-air piezoelectric structure, the system of equations is :

$$\begin{bmatrix} [K_{uu}] - \omega^2[M] & [K_{u\Phi}] \\ [K_{u\Phi}]^T & [K_{\Phi\Phi}] \end{bmatrix} \begin{bmatrix} U \\ \Phi \end{bmatrix} = \begin{bmatrix} F \\ -Q_p \end{bmatrix}$$

where **U** and **Φ** are the vectors containing the nodal values of the displacement field and of the electrical potential, **F** and $\mathbf{Q_p}$ contain the nodal values of the applied forces and of the electrical charges. $[K_{uu}]$ and [M] are the structure stiffness and mass matrices, $[K_{u\Phi}]$ and $[K_{\Phi\Phi}]$ are the piezoelectric and dielectric stiffness matrices. ω is the angular frequency.

In practical cases, the general system of equation has to be modified, depending upon the electrical boundary conditions and the type of applications [3]. First, the electrical potential vector **Φ** is partitioned into two parts, the applied electrical potential $\mathbf{\Phi_A}$, which is the same for all the nodes of the hot electrode and the vector $\mathbf{\Phi_I}$ which includes the electrical potential nodal values for all the inner nodes. The reference potential is assumed to be zero (grounded electrode). Similarly, $\mathbf{Q_p}$ can also be split into two parts. The nodal value associated with $\mathbf{\Phi_A}$ is equal to I/jω, where I is the current entering the hot electrode. The nodal values associated with the inner nodes are all equal to zero. Then, the system is simplified, after summing the lines corresponding to the hot electrode nodes and deleting those corresponding to the ground electrode.

If the transducer is loaded by an external impedance, the applied voltage $\mathbf{\Phi_A}$ is simply equal to IZ where Z is the complex electrical impedance to which each piezoelectric element is assumed to be coupled. Solving the simplified system of equations provides the displacement field and the electrical potential in all the structure. By varying the value of the external impedance Z, damping can be evaluated.

3.0 FIRST APPLICATION

The first application is a piezoelectric ring placed between two masses (Fig. 1). The piezoelectric ring is made of PZT4, inner radius of which is equal to 2 mm, its external radius is 5 mm and its length is 18 mm. The piezoelectric ring is placed between two masses, the weights of which are 2.272 kg each and is shunted by a passive resistance. A force is applied on one face of the system. This example is close to the example considered by Law *et al* [2]. With a view to having the value of the optimal resistance, an equivalent electrical circuit model [4] is used. The model describes the electrical behavior of piezoelectric material and determines the required optimal resistive load with a view to minimizing the acceleration on the opposite face.

Figure 2 presents the Frequency Response Function (*FRF*), which is the ratio between this acceleration and the applied force, calculated with the finite element method, as a function of the frequency. The *FRF* shows the damping capability of the system and is expressed in dB. Different values of the external resistance are considered. The lowest level is clearly obtained for $R = R_{opt}$, in the frequency band of interest.

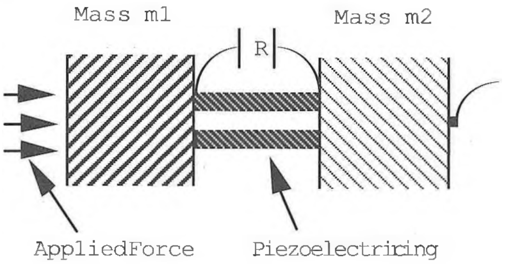

Fig. 1. Description of the system.

With a view to improving the model, it can be envisaged to shunt the piezoelectric material by a resistance and an inductance, the optimal values of which have to be determined. Experiments are under progress.

4.0 SECOND APPLICATION

The second application is a aluminum plate, clamped on a boundary and submitted to a force on the opposite side (Fig. 3). A piezoelectric plate is put on the plate. Its size and its position are determined with a view to maximizing the coupling factor. Then, the piezoelectric element is shunted by an external electrical circuit. In that case, with a view to finding the value of the optimal external impedance, the equivalent electrical circuit model is no more suitable but a numerical approach, using the modal decomposition, is used. Figure 4 presents the variations of the normalized displacement at the boundary of the plate as a function of the frequency, normalized to the first resonance frequency, with open circuit and with the external electrical circuit. It is clear that damping is observed, when the value of the external impedance is optimized. Experiments are under progress.

More generally, this approach could be used for any structure, containing a piezoelectric part, coupled to an external impedance [5]. Moreover, applications to sensor-actuator panels for underwater acoustic control could be foreseen [6].

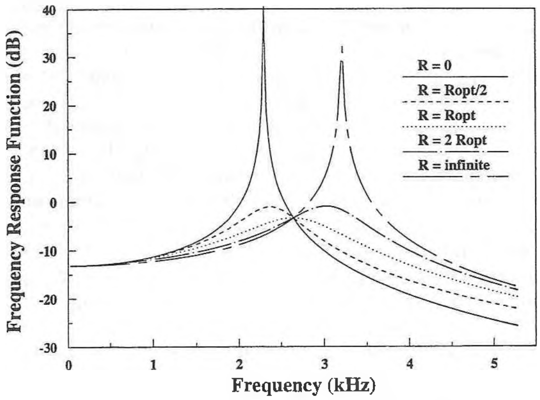

Fig. 2. Variations of the Frequency Response Function (*FRF*) in dB, as a function of frequency, for different values of the external resistance.

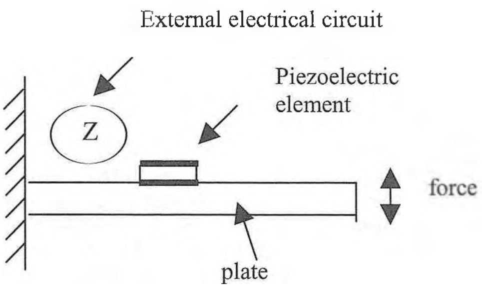

Fig. 3. Aluminum plate, with a piezoelectric element connected to an external electrical circuit.

5.0 REFERENCES

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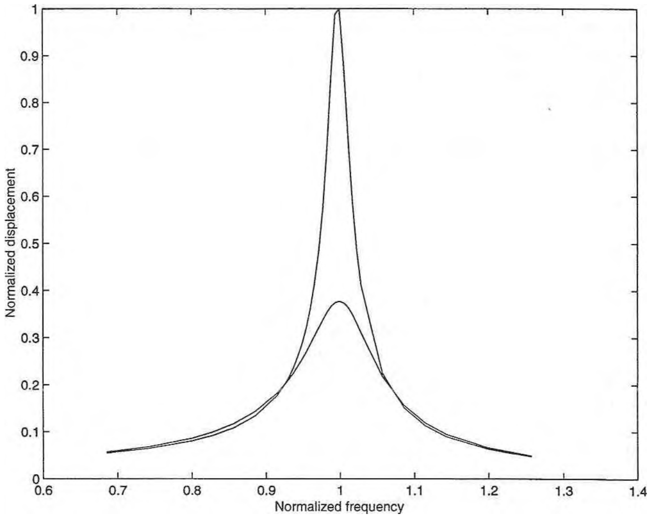

Fig. 4. Normalized displacement at the boundary of the aluminum plate, with open circuit and with the external electrical circuit, as a function of the normalized frequency.