Recursive Least-squares Algorithms with Improved Numerical Stability and Constrained Least-squares Algorithms for Multichannel Active Noise Control Systems

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1.0 Introduction

This paper deals with the convergence of adaptive FIR filters used for multichannel active noise control (ANC) systems [1]. In a recent paper [2], recursive least-squares (RLS) algorithms and fast-transversal-filter (FTF) algorithms were introduced for multichannel ANC. It was reported that these algorithms can greatly improve the convergence speed of ANC systems, compared to algorithms using steepest descent algorithms or their variants, as expected. However, numerical instability of the algorithms was an issue that needed to be resolved. This paper summarizes some work that has recently been done to address this numerical instability problem. For the full detailed description of the new proposed algorithms, the reader should refer to [3].

In this paper, extensions of stable realizations of recursive least-squares algorithms such as the inverse QR-RLS [4], the QR decomposition least-squares-lattice (QRD-LSL) [4] and the symmetry-preserving RLS algorithms [4] are first discussed for the specific problem of multichannel ANC. Constrained adaptive least-squares algorithms for multichannel ANC are discussed next. The need for these constrained algorithms is mostly in undetermined ANC systems (systems with more actuators than error sensors), where even the typically numerically stable realizations of RLS algorithms will not produce stable convergence, unless some noise is added to the input signals of the algorithms. The algorithms to be discussed use either the structure of the filtered-x LMS algorithm for ANC [1] (i.e. the "filtered-x structure") or the structure of the modified filtered-x LMS algorithm for ANC [6] (i.e. the "modified filtered-x structure"). The main difference between the two structures is that the modified filtered-x structure computes an estimate of the primary field signals and removes the effect of the plant delay [6]. As a consequence, higher adaptation gains can be used in the algorithms using the modified filtered-x structure.

2.0 Inverse QR-RLS algorithms for multichannel ANC systems

The QR-RLS and the inverse QR-RLS algorithms (also called square-root RLS algorithms) are known to have very good numerical stability, because they make use of matrix rotations with good numerical properties [4]. However, the QR-RLS algorithm does not explicitly computes the coefficients of the adaptive FIR filters. For multichannel ANC systems, this is an important issue. It is not sufficient to have an algorithm that produces an estimate of a target signal for each iteration of an algorithm, it is also required to have the knowledge of the adaptive filter coefficients. To obtain those coefficients from the QR-RLS algorithm, an inversion of a square-root inverse correlation matrix is required. This adds many operations to the computational load, and may reduce the stability of the solution if the matrix is ill-conditioned. At the opposite, the inverse QR-RLS algorithm computes explicitly the adaptive filter coefficients [4]. This is why this algorithm was chosen as the algorithm to be extended for multichannel ANC systems. An inverse QR-RLS algorithm for multichannel ANC systems was thus developed in [3] for both the filtered-x and the modified filtered-x structures, and the numerical stability of the algorithm will be discussed later in this paper.

3.0 QRD-LSL algorithms for multichannel ANC systems

The inverse QR-RLS algorithm for multichannel ANC systems of the previous section has a computational load proportional to the square of the number of adaptive filter coefficients. For adaptive FIR filters with a lot of coefficients, this computational load can become too high for real-time implementations. This is the motivation for developing "fast" RLS algorithms for multichannel ANC systems, where the computational load will increase linearly with the number of adaptive filter coefficients, not quadratically. A low computational realization is the fast-transversal-filters (FTF) algorithm, and a FTF algorithm for multichannel ANC was introduced in [2]. It is however well known that this algorithm suffers from numerical instability, and although it is possible to use a "rescue variable", it was found using simulations that for multichannel ANC systems this often lead to continuously re-initialized algorithms (and therefore a slower convergence). It is also not a trivial task to adjust the threshold for the "rescue variable", in particular for multichannel systems. There is thus a need for a numerically stable "fast" algorithm for multichannel ANC. The QR-decomposition least-squares-lattice (QRD-LSL) algorithm [4] is known to be a very numerically robust algorithm. It can be used to develop the least-squares-lattice (LSL) algorithms [4]. Some of those LSL algorithms also have a good numerical stability (in particular the versions with error feedback), but the stability of the QRD-LSL algorithm is typically better [4]. The QRD-LSL algorithm was thus extended for multichannel ANC systems in [3], and the numerical stability of the extended algorithm will be discussed later in this paper.

Since the QRD-LSL algorithm requires the knowledge of the "target" or "desired" signals, it can only be developed for ANC systems with the modified filtered-x structure. Also, the QRD-LSL provide auxiliary joint-process coefficients which are different from the time domain adaptive filter coefficients (i.e. the auxiliary coefficients are in a time-varying transformed basis). Therefore a time-varying inverse transformation from the auxiliary joint process coefficients to the time domain coefficients is required. Such an inverse transformation exists for the classical QRD-LSL algorithm [4], and it can be extended to the multichannel ANC case [3]. However, the inverse transform has a computational load proportional to the square of the number of adaptive filters coefficients, and the reason for developing the QRD-LSL for multichannel ANC systems was to avoid such a quadratic dependency. However, the inverse transform does not need to be computed on a sample by sample basis: it can be computed and applied at a reduced rate. The update of the multichannel joint-process auxiliary coefficients can still be done on a sample by sample basis, as in any recursive least-squares algorithm, it is just the update to the time-domain coefficients that occurs at a reduced rate. For example, if a forgetting factor of 0.999 is used in the algorithm (and therefore the "memory" of the algorithm has a time constant of 1000), then updating the coefficients every 10 or every 100 samples will not greatly affect the convergence performance or the tracking performance of the algorithm.

4.0 Symmetry-preserving recursive least-squares algorithm for multichannel ANC systems

Another approach to develop a numerically stable RLS algorithm for multichannel ANC is to use a simple symmetry-preserving approach [4]. This approach simply requires to compute the upper triangular part of the inverse time-averaged correlation matrix in the RLS algorithm, and then copy the values to the transposed positions in the lower triangular part. It is reported that this simple approach improves the numerical stability of the standard RLS algorithm [4]. It is straightforward to modify the RLS algorithm for multichannel ANC systems found in [2] to obtain a symmetry-preserving version. This is done in details in [3] for both the filtered-x and the modified filtered-x structures, and the numerical stability of the resulting algorithm will be discussed later in this paper.

5.0 Constrained least-squares algorithms for multichannel ANC systems

As mentioned in the introduction, in some cases multichannel ANC systems will be underdetermined, and in these cases unconstrained recursive least-squares algorithms may become numerically unstable unless some noise is added to the input signals of the algorithms. In [3], three constrained least-squares algorithms were developed for multichannel ANC systems. The different constraints were: the minimization of the frequency weighted actuator outputs power, the minimization of the adaptive filters coefficients squares, and a pseudo-inversion of the multichannel correlation matrix. Because of the constraint, the matrix inversion lemma is not directly applicable, and therefore these least-squares algorithms are not recursive like the algorithms of the previous sections. Also, the least-squares algorithms require the knowledge of the primary field signals, and therefore they are only applicable to the modified filtered-x structure. The reduce the high computational load required by the inversion of the multichannel correlation matrix, an option is to compute the inversion offline every 10, 100, or 1000 iterations (with preferably less iterations between updates than the time constant caused by the forgetting factor). The numerical stability of the constrained least-squares algorithms will be discussed in the next section.

6.0 Simulation results

In order to evaluate the numerical stability of the different algorithms, simulations were performed in [3] using a C program implementation, with a single precision floating point representation. The code from [7] was used for matrix inversions with LU decompositions and, in the case of the constrained least-squares algorithm with pseudo-inversion, for matrix pseudo-inversions with SVD decompositions. The simulations were performed for multichannel ANC systems with the modified filtered-x structure. Time domain and frequency domain characteristics of some of the transfer functions from which the simulations were performed can be found in [2]. Adaptive filters with 100 coefficients each were used in the simulations.

The first set of simulations was performed for a system with one reference sensor, two actuators and two error sensors (a 1-2-2 system). The algorithms tested for this first set of simulations were the RLS and FTF algorithms found in [2], and the new inverse QR-RLS, QRD-LSL and symmetry-preserving RLS algorithms. Table 1 compares the numerical stability of the different algorithms for multichannel ANC systems. From Table 1, it is clear that the RLS and FTF algorithms developed for multichannel ANC systems are numerically unstable, as previously reported in [2]. Although it is obtained from a slight modification to the numerically unstable RLS algorithm, the symmetry-preserving RLS algorithm was found to be stable (unless a very low value of forgetting factor such as 0.9 was used). The QRD-LSL algorithm was always stable and always produced a good performance in the transformed domain, but the simulations have shown that the inverse transformation required to compute the time-domain adaptive filters coefficients only produces a good convergence performance in the time-domain when the value of the forgetting factor was sufficiently high (such as 0.999 shown in Table 1). Finally, the inverse QR-RLS produced the best performance of all algorithms: it was always stable and always produced a good attenuation.

A second set of simulations was performed in [3], this time for an underdetermined 1-3-2 system. In this second set of simulations, the algorithms that were simulated were the recursive least-squares algorithms used in the first set of simulations and the constrained least-squares algorithms of Section 5. Table 2 summarizes the results of this second set of simulations. First of all, since the simulated system is underdetermined, the (unconstrained) recursive least-squares algorithms all suffered from numerical instability. At the opposite, all the constrained least-squares algorithms were numerically stable if a sufficient constraint was used (a good attenuation could still be achieved with the constraint). The update of the coefficients in these constrained least-squares algorithms was computed once for every 100 iterations, but this did not reduce much the convergence speed.

7.0 Conclusion

This paper discussed recursive least-squares algorithms with improved numerical properties and constrained least-squares algorithms for multichannel ANC systems. The choice of the most suitable algorithm for ANC will depend on the specific application. If the ANC system is underdetermined, then the constrained least-squares algorithms of Section 5 may be used. If the ANC system is not underdetermined, then the algorithms of Sections 2-4 may be used. In that case, the inverse QR-RLS algorithm has the best numerical properties, but it also has the highest computational load. The QRD-LSL algorithm has the lowest computational load [3], but it requires the use of a forgetting factor with a fairly high value (typically 0.999 or higher). At last, the symmetry-preserving RLS has the simplest software implementation (but not the lowest computational load). This may be a factor is some implementations.

References

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Multichannel ANC algorithm (forgetting factor 0.999)	Numerical stability/instability for the 1-2-2 system
RLS	unstable, 200 iterations
FTF (no rescue variable)	unstable, 27000 iterations
inverse QR-RLS	stable, 23 dB average attenuation
QRD-LSL	stable, 22 dB average attenuation
symmetry-preserving RLS	stable, 23 dB average attenuation

Table 1 Numerical behavior of the algorithms for the 1-2-2 system

Multichannel ANC algorithm (forgetting factor 0.999)	Numerical stability/instability for the 1-3-2 system
RLS	unstable, 200 iterations
FTF (no rescue variable)	unstable, 1300 iterations
inverse QR-RLS	unstable, 50000 iterations
QRD-LSL	unstable, 3000 iterations
symmetry-preserving RLS	unstable, 9000 iterations
least-squares constrained with actuators power	stable, >35 dB attenuation
least-squares constrained with coefficient squares	stable, >35 dB attenuation
least-squares using SVD pseudo-inverse	stable, >60 dB attenuation

Table 2 Numerical behavior of the algorithms for the 1-3-2 system