Inverse Characterization of the Geometrical Macroscopic Parameters of Porous Materials

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1. Introduction

This paper considers the inverse problem of determining acoustically the geometrical macroscopic parameters (geometrical tortuosity α_∞, viscous Λ and thermal Λ' characteristic lengths) of an open celled porous specimen using a standing wave tube. The unknown parameters can be identified from simple measurements of the normal specific acoustic impedance of the specimen. The inverse characterization scheme is based on the equivalent fluid model in which the solid frame is assumed to be rigid, i.e. motionless. The identification of the parameters is performed over the frequency range [50-2000 Hz]. The test specimen is backed by the rigid impervious end of the tube and held tight on its contour. This set of boundary conditions, especially under acoustical excitations, tends the frame to be motionless.

In the following, a description of the equivalent fluid model is firstly presented. Secondly, the inverse problem strategy is briefly discussed. Thirdly, inverse characterization results on a real porous specimen are given. Finally, to confirm the validity of the approach, numerical predictions using the inversely identified parameters are compared to the measured sound absorption coefficient of the specimen for different thicknesses.

2. Equivalent fluid model

Allard [1] has shown that the acoustic behavior of open celled porous materials can be well described with the use of the Biot theory. When excited by acoustical waves, frames of these materials behave approximately as acoustically rigid (motionless) over a wide range of frequencies [1,2]. For such a case, the porous material can be replaced on a macroscopic scale by an equivalent fluid of effective density and effective bulk modulus occupying a proportion ϕ of the volume of the porous material. In the widely used equivalent fluid model of Johnson-Champoux-Allard[1], these effective quantities depend on five macroscopic parameters of the porous medium: the flow resistivity σ, the porosity ϕ, the tortuosity α_∞, and the viscous Λ and thermal Λ' characteristic lengths. According to this model, expressions for the and have been suggested [3]:

$$\tilde{\rho}(\omega) = \rho_0\alpha_\infty\left(1+\frac{\sigma\phi}{j\omega\alpha_\infty\rho_0}G_J(\omega)\right) \tag{1}$$

$$\tilde{K}(\omega) = \frac{\gamma P_0}{\gamma-(\gamma-1)\left(1+\frac{H'}{j2\omega}G_{J'}(\omega)\right)^{-1}} \tag{2}$$

where $G_J(\omega)$ and $G_{J'}(\omega)$ are respectively viscous and thermal correction factors defined as:

$$G_J(\omega) = \left(1+j\frac{\omega}{H}\right)^{1/2} \tag{3}$$

$$G_{J'}(\omega) = \left(1+j\frac{\omega}{H'}\right)^{1/2} \tag{4}$$

with

$$H = \frac{\sigma^2\Lambda^2\phi^2}{4\alpha_\infty^2\eta\rho_0} \quad \text{and} \quad H' = \frac{16\eta}{B^2\Lambda'^2\rho_0} \tag{5}$$

where ω is the angular frequency, ρ_0, η, γ, and B^2 are the density, the dynamic viscosity, the ratio of the specific heats and the Prandtl number of the air. The tilde (~) indicates that the associated variable is complex-valued and frequency dependent.

For a porous specimen of thickness d, backed by a rigid impervious wall, its specific acoustic surface impedance is:

$$Z_s = -j\frac{Z_c}{\rho_o c_o}\cot(k\,d)/\phi \tag{6}$$

where Z_c and k are the characteristic impedance and the complex wave number of the porous specimen, respectively. They are related to the effective properties of the porous medium by:

$$Z_c = (\tilde{\rho}(\omega)\tilde{K}(\omega))^{1/2} \tag{7}$$

$$k = \omega\left(\frac{\tilde{\rho}(\omega)}{\tilde{K}(\omega)}\right)^{1/2} \tag{8}$$

3. Inverse problem strategy

In this work is presented application of an acoustic inverse problem (i.e. acoustic experimental/numerical procedure), for the characterization of the three geometrical macroscopic parameters (α_∞, Λ, Λ') of a porous specimen. The proposed strategy is based on the standing wave tube measurement and on the acoustical model (using Eq. 6) of the specific surface impedance Z_s of the specimen. The presented inverse scheme can be interpreted as a nonlinear optimization problem wherein the cost function is defined as the difference between measured and predicted values of the specific acoustic impedance. Using the nonlinear least squares approach, the cost function to be minimized is defined as:

$$R(\mathbf{a}) = \frac{1}{2}\,\|F(\mathbf{a})\|_2^2 = \frac{1}{2}\sum_{i=1}^{N}(F_i(\mathbf{a}))^2 \tag{9}$$

such that, $lb \le \mathbf{a} \le ub$ with,

$$F_i(\mathbf{a}) = |Z_{sR}(\omega_i;\mathbf{a}) + j\,Z_{sI}(\omega_i;\mathbf{a})| - |Z_{iR} + j\,Z_{iI}| \tag{10}$$

where lb is the vector of lower bounds, and ub is the vector of upper bounds. Z_{iR} and Z_{iI} are respectively the measured real and imaginary parts of the specific surface impedance at the i-th angular frequency ω_i. $Z_{sR}(\omega_i,\mathbf{a})$ and $Z_{sR}(\omega_i,\mathbf{a})$ are the corresponding numerical prediction at the same angular frequency for the unknown adjusted parametric vector $\mathbf{a} = \{\alpha_\infty, \Lambda, \Lambda'\}^T$.

Eq.10 represents a set of N equations, where N is the number of measured data which is arbitrarily large and only depends on the sampling of the frequency domain.

A number of numerical tests have been performed and the results were used like a prior knowledge of how to set up an adequate statistical optimization algorithm [4] to solve such a difficult problem in the shortest time. In the second step, the final set up parameters estimation algorithm was applied for evaluation of the geometrical parameters from real experimental data.

4. Application of the inverse strategy

To start the inverse characterization strategy, standing wave tube measurements of the specific surface impedance for three specimens taken from an elastic foam were performed. The specimens were 24.67, 24.70, and 24.73-mm thick, respectively, with a diameter of 99.8-mm. The diameter of the specimens is slightly greater than the diameter of the tube, so that the specimens were held tight to reduce frame motion. The measurements were done with a B&K 4206 impedance tube. Figure 1 reports the results. The results indicate that the foam from which the specimens were taken seems homogeneous in its material properties, at least at the scale of the specimens.

For the three specimens, a direct characterization of the flow resistivity, open porosity, foam density, and elastic properties were done using LCMA facilities [5]. The mean values of these properties are given in Table 1.

Using the inverse characterization strategy, discussed in the previous section, with the acoustical model (Eq. 6) and the measured specific surface impedance (Fig. 1), the three geometrical properties (α_∞, Λ, Λ') are estimated. Their numerical values are given in Table 2 [more details on the algorithm will be presented at the conference].

5. Numerical simulations

To partly verify the validity of the inverse strategy, numerical simulations using the parameters given in Tables 1 and 2 are compared to standing wave tube measurements of the sound absorption coefficient for three different thicknesses of the foam. The numerical simulations are done with MNS/Nova™[6] which is based on the Biot and Johnson-Champoux-Allard models. The rigid-frame limit of the Biot model was used.

Figure 2 presents these comparisons. It is noted that the numerical simulations are in very good agreement with the measured sound absorption coefficients. Since the estimated parameters, based only on the 24.70-mm test specimens (Fig. 1), leads to fine predictions for two other thicknesses, it may be concluded that they are not just "fudge factor" but physical.

Table 1 - Direct measurement of some foam parameters

ϕ	Porosity	0.96
σ	Flow resistivity (Ns/m^4)	4971
ρ_1	Bulk density (kg/m^3)	21.66
E	Young's or elastic modulus (Pa or N/m^2)	46 300
ν	Poisson's ratio	0.37
η	Damping loss factor	0.135

Table 2 - Inverse characterization of the geometrical parameters

α_∞	Geometrical tortuosity	1.25
Λ	Viscous characteristic lengths (10^{-6} m or µm)	105.8
Λ'	Thermal characteristic lengths (10^{-6} m or µm)	339.1

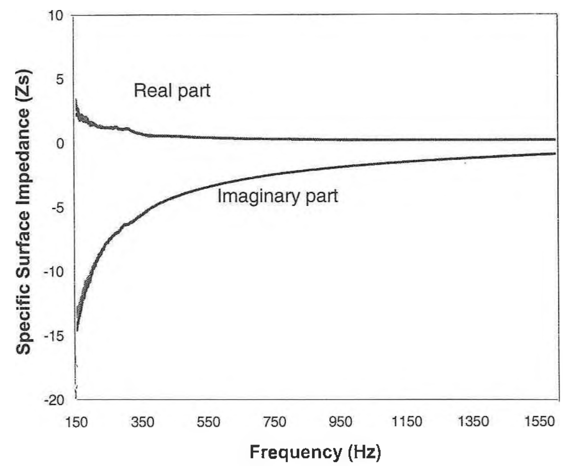

Figure 1 - Standing wave tube measurements of the specific surface impedance for three specimens of the same material. The specimens are 24.67, 24.70, and 24.7-mm thick.

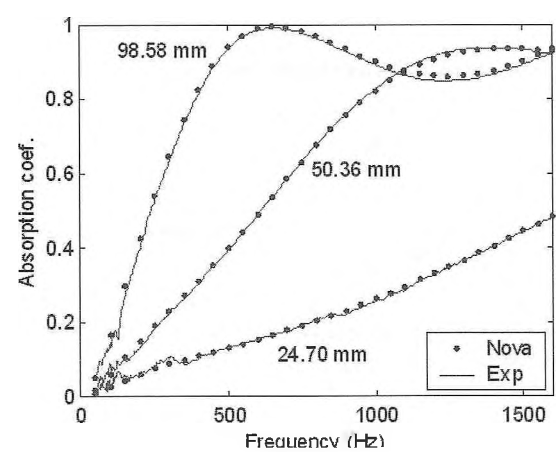

Figure 2 - Comparison between numerical predictions using properties given in Tables 1 and 2 and standing wave tube measurements in terms of sound absorption coefficient for three thicknesses of the tested foam.

6. References

[1] J. F. Allard., Propagation of Sound in Porous Media: Modeling Sound Absorbing Materials (Elsevier Applied Science, New York, 1994).

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[4] Bäck, T.,Evolutionary Algorithms in Theory and Practice, (University Press, Oxford, 1995), 314p.

[5] R. Panneton, Acoustic Materials Characterization Lab, SEA Tech-Club 2000, 2nd meeting, Troy, MI.

[6] MNS/Nova™, version 1.0. http://www.mecanum.com