1. Introduction

Porous media, used as passive absorbers, are usually considered as single porosity material. Their macroscopic properties (dynamic permeability and compressibility) can be deduced from the knowledge of one elementary representative volume (ERV), defined at the microscopic scale.

Physics and absorption properties of double porosity media can be significantly different, because two interconnected networks of very different sizes, and then permeabilities coexist in the system.

In this paper we clarify the coupling effects that occurred in these systems according to the contrast of static permeability in the two fluid networks.

Two macroscopic descriptions are proposed and compared to experimental measurements performed on artificial double porosity media. Then, the advantages of such systems are underlined.

2. Theory

A double porosity medium can be seen as a porous medium with a micro-porous solid skeleton. Then, three scales are necessary to describe the material (Fig. 1): the macroscale for which the medium appears to be homogeneous. The wavelength in the material allows to estimate this scale. The mesoscale is defined from the pores and microporous domains, and is characterized by the size \( l_m \). Last, the microscale is related to the micropores and the solid skeleton (assumed rigid), and is characterized by the size \( l_M \).

![Fig.1 The three scales in double porosity media](image)

The dynamic behaviors of such a system can be successfully worked out using the rigorous homogenization technique of multi-scales periodic media [1]. In that way, the first step consists in understanding the consequences of the double separation of scales and estimating parameters, and couplings, governing waves propagation in the medium.

Two main behaviors, depending on the contrast of static permeability between the pores and the micropores, can be identified [2][3], and are now briefly described.

Assuming that waves propagation in the pores is not strongly affected by the presence of the micropores, the wavelength (neglecting thermal effects) in the material can be estimated by the one in the pores. Using asymptotic developments at low and high frequencies, it comes:

\[
|\lambda_p| = \frac{P_0}{\rho_0} \frac{1}{\sqrt{2\pi P}} \quad \text{for } \omega \ll \omega_{vp} \quad (1)
\]

\[
|\lambda_m| = \frac{P_0}{\rho_0} \frac{1}{\sqrt{2\pi P}} \quad \text{for } \omega \gg \omega_{vp} \quad (2)
\]

with \( \omega_{vp} = (\eta / \rho_0)^{1/2} \), the viscous frequencies in the pores.

\( P_0, \eta, \) and \( \rho_0 \) are respectively the static pressure, the viscosity and the density of air.

An analog estimation can be obtained for the wavelength in the micropores (subscript \( m \) replaced by \( p \)).

These estimations are very important in order to predict the variation range of the pressure in the fluid networks \( O(\sqrt{l} / 2\pi) \), and the ratio between local velocities.

Case 1: low permeability contrast

In this case, the difference between \( l_m \), and \( l_M \) is small enough so that \( (|\lambda_p| / 2\pi) \) and \( (|\lambda_m| / 2\pi) \) are always greater than \( l_m \), the characteristic size of the “meso-heterogeneities”. Then, it can be shown that the pressure is uniform in the double porosity system, meaning that it varies at the macroscopic scale \( x \) space variable. It results a total coupling of the pores and the micropores in terms of compressibility, and permeability when the frequency is greater than \( \omega_{vm} \). General macroscopic equations obtained in this case are:

**Compressibility:**

\[
j \omega \left( \frac{1}{K_m(\omega)} + \frac{1}{K_p(\omega)} \right) p(x) + \nabla \cdot \nabla \phi = 0 \quad (3)
\]

**Dynamic flow:**

\[
\nabla \cdot \phi = \frac{\Pi_m(\omega) }{\eta} \nabla_p \cdot p(x) \quad (4)
\]

In these equations, acoustical variables are the first terms of the asymptotic developments used in the homogenization process. Eq. (3) shows clearly how the equivalent compressibility of the system can be simply expressed by means of \( K_m \) and \( K_p \), respectively the equivalent bulk modulus of the porous medium (without microporosity), and of the microporous material. \( \phi_p \) is the porosity of the network of pores.

Regarding the macroscopic dynamic flow (4), we showed that it can be written in the form of a “dynamic Darcy law”. The equivalent permeability \( \Pi_m \) depends on the microporous medium dynamic equivalent permeability \( \Pi_m \), and on the meso-structure. Analytic expressions can be found for simple meso-structures.

Case 2: strong permeability contrast [4]

Now, it is supposed that the characteristic size \( l_m \) is small enough, so that the \( |\lambda_m| / 2\pi \) can be of the same order as \( l_M \).
being large compared to $|\lambda_n|$. This situation can only be raised if waves are diffusive in the micropores (below $\omega_{cm}$). This involves that pressure in the microporous domain varies at the mesoscale (space variable $y$) and we have:

$$\Delta_s p_m(x,y) - \int_0^{\omega_{cm}} \frac{\phi_m \Pi_m(0)}{\omega} p_m(x,y) = 0 \text{ in } \Omega_{sp}$$

(5)

$\Pi_m(0)$ is the intrinsic static permeability of the microporous medium. Equation (5) is bounded by the macroscopic pressure field in the pores ($p_p(x)$). The problem is analog to the thermal diffusion problem in porous media and show that $p_p$ is linearly related to $p_m$. The phenomenon raises a new characteristic frequency ($\omega_g = p_p \eta / \phi_m \Pi_m(0)$). Moreover, the average pressure in the microporous structure can be written in the form:

$$\langle p_m \rangle = F(\omega / \omega_g) p_p$$

(6)

where $F$, complex, denotes the partial coupling between pores and micropores. Its modulus varies between 1, and 0 from low to high frequencies. It only depends on the meso-geometry, and can be expressed with simple semi-phenomenological functions [3] for general descriptions.

Finally, the macroscopic mass equation giving the equivalent compressibility (with thermal effects) of the system is:

$$\int_0^{\omega_{cm}} \frac{1}{K_s(\omega)} \left[ \frac{\phi_m}{\omega} \frac{p_p(x)}{p_m(x,y)} \right] \omega \eta \phi_m \Pi_m(0) = 0 \text{ in } \Omega_{sp}$$

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(7)

Regarding the macroscopic permeability, it is given, in this case, by the equivalent permeability of the porous medium (without microporosity), flow in the micropores being negligible at the first order.

3. Experimental results and discussion

Artificial double porosity materials have been built by perforating (pores network) microporous rock wool panels (5.75 cm thick.). The cylindrical holes are circular (radius R) and placed in square lattice, perpendicular to the panels’ faces (Fig.2). This configuration allows to determine a maximum of parameters for the model and the material is easy to realize.

Measurements have been made using standing waves tubes, in the direction of perforations. The rock wool material have been chosen for its high resistivity ($\sigma = 135000 \text{ Nm}^{-2} \text{s}^{-1}$) with the aim to observe pressure diffusion effects. First, results show that the “total coupling” model is not able to describe the absorption coefficient of the double porosity medium (Fig.3). However, a good agreement can be obtained with the “partial coupling” model, what demonstrates that pressure is not uniform in the system at the mesoscale. This phenomenon brings additional dissipation, and the absorption of the initial rock wool panels is greatly improved on a wide range of frequencies by creating a second network of pores. It is to be noticed, that for this low additional porosity ($\phi_p = 13\%$), taking flow in the microporous medium into account improves the agreement with measurements. In this case, a simple and exact expression can be found for the equivalent dynamic permeability in the perforations direction:

$$\pi_m(\omega) = (1 - \phi_p) \pi_m(\omega) + \phi_p \eta / j \omega p_p$$

(8)

In this last equation, viscous dissipation is neglected in the perforations, considering their size. A second series of measurements (Fig.4) have been performed on the same material, with smaller perforations, in order to modify $\phi_p$. The results show how the performances of the material can be easily adjusted to obtain low frequencies efficient materials.

4. References


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