Acoustic Absorption Of Non-Homogeneous Poro-Elastic Materials

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1. Introduction

Porous materials offer a poor acoustic absorption performance at low frequencies. A way to improve the sound absorption of these materials is to investigate 3-D complex configurations with the finite elements method. The main objective of this paper is to propose an accurate model to evaluate the acoustic absorption performance of configurations consisting of a non-homogeneous porous material, made up from porous-elastic patches, bonded onto the hard termination of a semi-infinite rectangular wave-guide. Each patch is correctly described in the context of the Biot theory [1], and the coupling between the porous material and the wave-guide is accounted for explicitly using the modal behavior of the wave-guide. A power balance approach is then used to evaluate the performance of the porous material in terms of its absorption coefficient.

2. Theoretical background

The finite element model associated to the porous-elastic media is based on the mixed displacement-pressure (u, p) formulation of Biot's poro-elasticity equations [2]. A modified weak integral form of these equations has the advantage to depict the boundary terms in a suitable form for the application of the coupling conditions with other media [3]. In this weak integral form, the boundary conditions terms of the porous media are:

$$\int_{\delta\Omega_p} (\underline{\underline{\tilde{\sigma}}}'(\underline{u})\cdot\underline{n})\cdot\delta\underline{u}\,dS + \int_{\delta\Omega_p} h(U_n - u_n)\,\delta p\,dS$$

where $\delta\Omega_p$ refers to the boundary surface of the porous-elastic domain Ω_p, $\underline{u}$ and p are the solid phase displacement vector and the interstitial pressure of the porous-elastic medium, respectively; $\delta\underline{u}$ and δp refer to their admissible variation, respectively; $\underline{n}$ denotes the unit normal vector external to the bounding surface; $\underline{\underline{\tilde{\sigma}}}'$ is the total stress tensor of the material; U_n and u_n refer to the normal component of the solid and fluid macroscopic displacement vectors, respectively, and h stands for the porosity of the material.

Using the modal behavior of the wave-guide, these boundary coupling conditions between the porous-elastic material and the wave guide can be expressed in terms of a classic coupling matrix, a radiation admittance and a blocked-pressure loading [4].

With a particular choice of admissible functions, the weak integral form provides the following power balance equation:

$$\Pi^s_{elas} + \Pi^s_{iner} + \Pi^f_{elas} + \Pi^f_{iner} + \Pi^{fs}_{coup} + \Pi^f_{exc} = 0$$

where Π^s_{elas}, Π^s_{iner} represent the power developed by the internal and inertia forces in the solid-phase in vacuo, respectively; Π^f_{elas}, Π^f_{iner} represent the power developed by the internal and inertia forces in the interstitial fluid, respectively; Π^{fs}_{comp} represents the power exchanged between the two phases; and Π^f_{exc} represents the power developed by external loading.
The time-averaged power dissipated within the porous medium can be subdivided into contributions from powers dissipated through structural damping of the skeleton, viscous and thermal effects:

$$\Pi_{diss} = \Pi^s_{diss} + \Pi^v_{diss} + \Pi^t_{diss}$$

The power Π^s_{diss} dissipated through structural damping is obtained from Π^s_{elas}, the power Π^v_{diss} dissipated through viscous effects is obtained from $\Pi^s_{iner} + \Pi^f_{iner} + \Pi^{fs}_{coup}$, and the power Π^t_{diss} dissipated through viscous effects is obtained from Π^f_{elas}. So, the total dissipated power and its components can be calculated for each element at a post-processing stage.

To characterize the absorption performance of the 3D studied patchworks, one defines the power absorption coefficient:

$$\alpha = \frac{\Pi_{diss}}{\Pi_{inc}}$$

where Π_{inc} is the incident power. If the excitation in the wave guide is a plane wave of amplitude p_0, this incident power is given by:

$$\Pi_{inc} = \frac{S|p_0|}{2\rho_0}$$

where S is the cross-section of the wave guide, ρ_0 and c_0 refer to the density and the velocity of the air in the wave guide, respectively.

3. Results

An experimental validation has been performed in the case of a macro-perforated material, which consists in a mineral wool with periodic holes containing air (called macro-pores) [5]. Such a material is referred to as a *double porosity* material. It consists in a periodic lattice made up of several periods of a generic cell which is a square mineral wool sample with dimensions $L \times L$ with a center square hole with dimensions $a \times a$. The macro-porosity ϕ_p corresponding to this cell is defined by:

$$\phi_p = \frac{a^2}{L^2}$$

Figure 1 shows the comparison between simulation and measurement for a macro-perforated sample whose the macro-porosity is 0.11. Excellent agreement is found, the numerical model reproduces the two biggest peaks. It proves the validity and increases the confidence level of the proposed method.

Next, a numerical simulation is shown to depict the absorption performance of a non-homogeneous material. Two configurations are considered. The first is a double-layer material made up from a 5 cm thick layer of a rock-wool and a 1 cm thick layer of a rigid glass wool. The rock-wool is bonded onto the rigid termination of a wave-guide. The second configuration is a non-homogeneous layer made with 6 cm of the rock-wool material in which the rigid glass wool is randomly distributed in the form of cubic cells. The ratio of the volume occupied by the two materials is kept constants in the two configurations. Figure 2 shows that the absorption performance is better with the non-homogeneous configuration. This result confirms that the different patches do interact, and that a non-homogeneous configuration increases the acoustic absorption.

Conclusion

The absorption coefficient of non-homogeneous porous-elastic layers has been predicted from a 3D numerical model where each patch is modeled with the Biot theory. An experimental validation has been presented in the case of a macro-perforated material, and it proved the accuracy of the presented model. Also, it has been proved that non-homogeneous layer has a better sound absorption than a multi-layer made of homogeneous materials.

References

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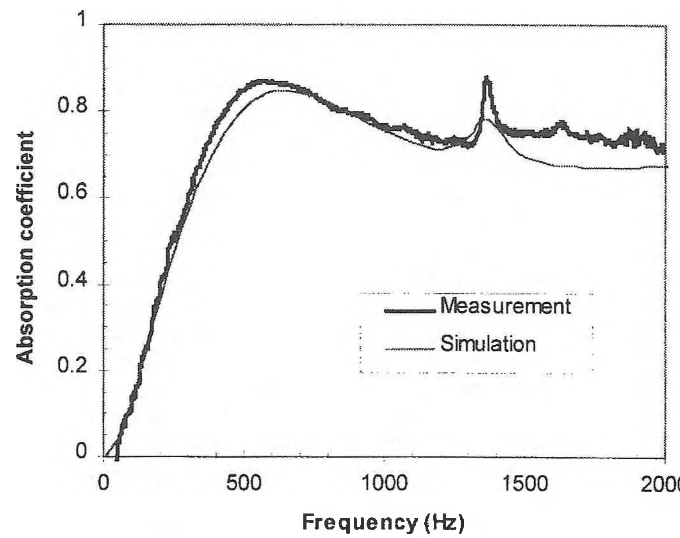

Figure 1. Comparison between prediction and measurements for a double porosity material.

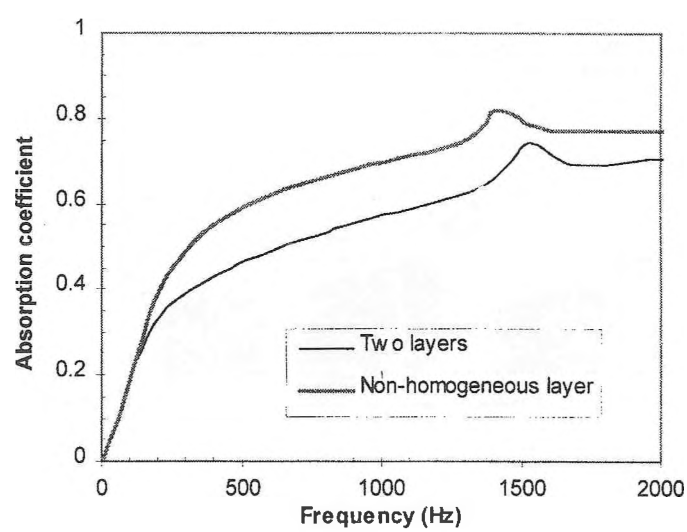

Figure 2. Comparison between a double layer and an equivalent non-homogeneous layer.