Wave Compatibility Condition: An Alternative For Vibro-Acoustic Problems In Medium Frequency Range

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Dealing with vibro-acoustic problems brings to the choice of a method to solve them. Those methods are usually classified in two categories: low and high frequency ranges. This is justified by the reachable or needed precision to achieve (versus the numerical cost).
If we want to keep the easiness of the FE-based or modal methods in writing the equations of the problem, we need to reduce the numerical size of the problem. For that purpose, we can see the boundary element methods and modal sampling ones as a great improvement.
In this paper, let's see how describing the primal and the dual fields on a very particular set of variables leads to straight-forwardly written equations along the boundaries. Moreover, on a rather simple generic vibro-acoustic problem, we will be able to derive the coupling equations in an easily understandable way.

Choice of the variables

Primal and dual local patterns are expanded on a basis of plane- or cylindrical-wavelike functions. These functions are selected among the solutions of the local equation of motion. The local variables are then the amplitudes $\{a_j\}_j$ of these functions.
In order to ensure the wave compatibility from one domain to an other when dealing with their coupling, we introduce two intermediate sets of boundary variables : $\{d_j\}_j$ and $\{F_j\}_j$. These variables are the trace of the local patterns projected, along the boundary on a Fourier series-like basis.

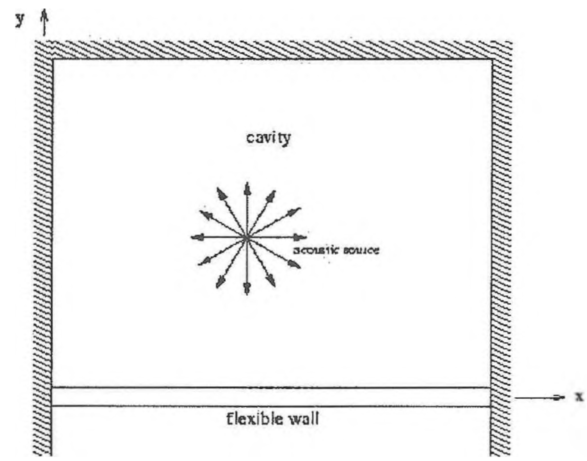
$$d_i = P_{1,ij}\, a_j$$

$$F_i = P_{2,ij}\, a_j$$

with P_1 and P_2, operators of projection.
The equation of coupling are then derived in term of boundary values; the Fourier-series decomposition allows us to write the coupling in a straight forward manner. The whole problem is then solved along the boundaries of the domains through the variables $\{d_i, F_i\}$ which are compatible with the description of wave propagation : the local primal and dual fields can be reconstructed thanks to their projections on these boundaries.
Therefore the modal behavior can be approached thanks to those boundary generalized values, whereas the propagative aspect is induced by the choice of the field variables. And the Wave Compatibility Condition (W.C.C.) ensures the connection between the two representations.

Vibro-acoustic example

Let's study the response to a point harmonic acoustic excitation of a rigid walled cavity, except on one side. This problem is considered as bi-dimensional, thus the flexible part will be described as a beam loaded with a continuous 2D field of pressure.



The number of generalized variables, N+1, for a portion (or face) of the boundary is given by the highest apparent number of wave lengths ,N , along that portion. Then, the total number of variables is limited to (N+1) x (element's number of faces).
We divide the domain in sub-elements so that the point exciting source can be located on the boundary of one of those elements. The excitation is then taken into account in the force continuity equations through this interface. In our case, if we divide the cavity in triangular elements, we have 3x(N+1) variables per element.
The boundary generalized values are the coefficients of a Fourier series which represent the projections of each wave onto the boundary of the sub-domain under scrutiny.
The fields of pressure p and the normal velocity v are described as introduced in the first part. The number of waves used in the basis is limited to 3N+3 per triangle sub-domain.

$$p(\vec{x}) = a_j \Phi_j(\vec{x})$$

$$v(\vec{x}) = i\rho\omega\, a_j \nabla\Phi_j(\vec{x}).\vec{n}$$

with j = 1, ... 3N+3.
The boundary generalized values $\{d_i, F_i\}$ are the amplitudes of the waves' projections on this very boundary.

$$p(s) = F^a{}_j G^a{}_j(s)$$

$$v(s) = i\omega\, d^a{}_j D^a{}_j(s)$$

with s the curvilinear variable, G^a_j Fourier function (sinus, cosinus) or dirac function located at the corner between two faces, D^a j Fourier function or linear interpolating function associated to one corner.
As to the beam, the transversal displacement w and the shear stress t are also expressed in the same way: the wave description

is used for the field patterns and the generalized-variable description is used along the boundaries:

$$t(s) = F^b{}_j G^b{}_j(s)$$

$$w(s) = d^b{}_j D^b{}_j(s)$$

Example of a loaded beam

In order to deal with the coupling of the flexible wall (the beam) and the cavity, we must first study the response of a clamped-clamped beam to an harmonic continuous loading.

The equation of motion gives the relation between the transverse displacement $w(x)$ and the continuous loading $f(x)$.

$$w^{(4)} - k^4 w = f(x) \qquad (1)$$

The clamped boundary conditions at the ends of the beam define a set of equations to be satisfied by the integrative constants. According to the type coupling here, $f(x)$ takes the form :

$$f(x) = F_C \cos(\bar{k}\,x) \text{ or } F_S \sin(\bar{k}\,x) \qquad (2)$$

with $\bar{k} = \frac{2\pi m}{L}$, L the length of the beam and $(F_C, F_S) \in R^{*2}$

Actually, $\bar{k}$ is linked to k by a relation involving the hysteretic damping of the beam, named η :

$$k = \bar{k}(1 - i\eta)$$

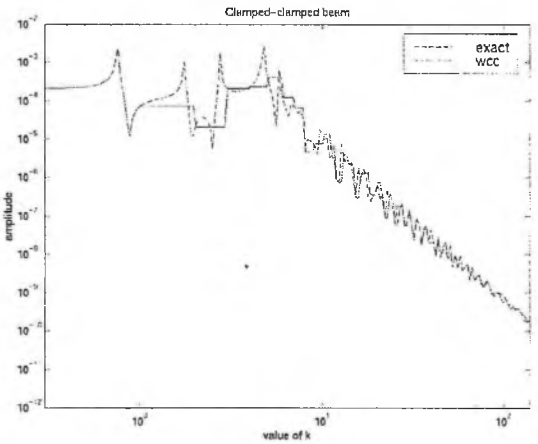
Solving (1) with (2) we obtain the response in amplitude as drawn on the following figure.

For a continuous variation of $\bar{k}$, we have drawn the exact response of the beam to time and space harmonic loadings. Yet, the WCC response of the same beam is observed by using the following approximation :

$$k \approx \frac{2\pi m}{L}(1 - i\eta)$$

with m, an integer number and

$$\frac{2\pi m}{L} \le \Re(k) < \frac{2\pi(m+1)}{L}$$



Finally, the WCC coupling with a bi-dimensional neighboring domain leads us to choose :

$$\bar{k} = \frac{2\pi m}{L} \text{ and } k = \frac{2\pi m}{L}(1 - i\eta)$$

By this mean, we are able to connect the solution of the loaded beam to the kind of coupling we use between the beam and the cavity (through Fourier functions).

The 3 different types of responses are compared on the figure at the bottom of this page. All the caracteristics of the beam are set equal to 1, the loading is in cosinus with 5 period along the length of the beam. The observation point is at 3/10 of the length from the end of the beam.

The step-like variations are due to the sharp variations of $\bar{k}$ when the frequency changes. Such effect becomes negligeable as the frequency rises up to medium and high ranges.

Coupling equations

We use this example to derive the coupling equations for the vibro-acoustic problem with $F_{C \text{ or } S} = F^b{}_m$. Each boundary variable is associated to a solution for the beam just like in the previous part. The final solution is obtained by adding all those single solutions.

The coupling between the flexible wall and the cavity, whatever dimension they might be is obtained by expressing their wave compatibility conditions through the set of Fourier variables defined at their interface $\{F^a{}_j, d^a{}_j\}$ and $\{F^b{}_j, d^b{}_j\}$.

Since we use the same kind of functions to describe the primal and the dual patterns along the interface (between the flexible wall and the cavity), the coupling equations are obtained by writing the continuity of the boundary variables from one domain to another. These equations represent the Wave Compatibility Condition.

Conclusion

The method presented in this paper allows us to solve vibro-acoustic problems in a straight-forward.

Once described on a local propagative basis, the primal and the dual fields are projected on the boundaries of each sub-domains where the coupling equations are derived thanks to a Fourier-like writing of the projected patterns.

This method keeps some advantages of so-called low frequencies ones and remains applicable to medium and high frequency ranges thanks to the condensed description of the primal and the dual fields.

References

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