A Hybrid Approach to The Mid-Fferquency Problem

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Overview

The response of a structural-acoustic system in the mid-frequency range typically consists of both long and short wavelength behavior. Modeling the short-wavelength behavior deterministically is usually computationally prohibitive and structural-acoustic techniques such as statistical energy analysis (SEA) are often adopted. However, SEA cannot adequately capture the long-wavelength global behavior of the system. Recent work aimed at addressing the mid-frequency problem has led to the development of a hybrid approach [1] based on a wavenumber partitioning scheme. This paper provides a brief overview of the approach.

1. Introduction

Consider the frame-plate structure illustrated in Figure 1. The structure consists of a stiff beam framework with two bays. A thin flexible plate has been inserted into one of the bays and the structure is excited by a point force applied to the framework as indicated.

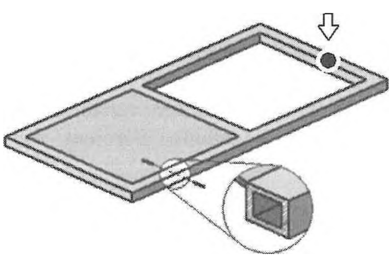

Figure 1. Frame structure typifying behavior of a system in the mid-frequency range.

The response of such a structure typically consists of a mix of both long and short-wavelength behavior over the frequency range of interest and is representative of a much wider class of mid-frequency problems. Such problems include the response of aerospace and automotive structures which contain stiffening frames, shells and enclosed acoustic cavities with disparate modal densities. The response of the framework in the previous example is dominated by long wavelength global behavior, while the response of the plate is dominated by short wavelength local behavior.

One approach to analyzing the dynamic behavior of the structure would be to model the interactions between the various beam and plate subsystems using an SEA model. However, the beam subsystems are typically strongly coupled and have relatively few interacting modes; their response therefore tends to be dominated by global rather than local dynamic behavior. In such circumstances SEA tends to overestimate the mean response due to coherence effects [2] and doesn't capture the resonant variations in the framework response.

One might therefore attempt to analyze the dynamic behavior of the structure using an FE model. However, the high modal density of the plate subsystem complicates such an analysis. Much of the computational effort involved in creating the FE model and solving the global eigenproblem is associated with capturing the short wavelength local behavior of the plate. Even if this computational expense is affordable one finds that the short wavelength behavior is also very sensitive to perturbations in the properties of the plate. One is then uncertain as to whether the predicted dynamic interactions between the plate and the framework are representative of those that occur in nominally identical structures.

It is therefore natural to question whether one can perform a hybrid analysis which combines FE and SEA in order to address the mid-frequency problem. This is the motivation behind the Resound method described in [1]. In Resound the subsystems in a system are partitioned into those that exhibit long wavelength global behavior (for example the framework in the previous example) and those that exhibit short wavelength local behavior (the plate). The latter are referred to as fuzzy subsystems. The long wavelength global behavior is then modeled deterministically using FE while the short wavelength local behavior is modeled statistically using SEA. There is clearly an interaction between the two partitions of the model and this interaction is fully accounted for with the calculation of various fuzzy coupling terms.

2. Local and global basis functions

The local and global partitioning described in the previous section is an important part of the Resound approach and merits further discussion. In general, a lumped-parameter model of a structural-acoustic system can always be obtained by expressing the response in terms of a finite number of basis functions. Equations of motion are then formulated using Lagrange's equations or Hamilton's principle. In Resound a distinction is made between short wavelength local basis functions (defined over the various fuzzy subsystems) and long wavelength global basis functions (defined over the whole system). The global basis functions are chosen so that they provide a good basis with which to describe the long wavelength global deformation of the system. Similarly, the local basis functions are chosen so that they provide a good basis with which to describe the local short wavelength behavior of the various fuzzy subsystems.

The global basis functions may be obtained by suppressing the local dynamic behavior in a (coarsely meshed) FE model. A convenient way to achieve this suppression is to apply Guyan reduction [3] to the interior degrees of freedom associated with each fuzzy subsystem. Additional techniques have also been developed to suppress individual wavefields within a FE model [5]. The local basis functions are then taken to be the local component modes associated with each fuzzy subsystem. The local component modes are never computed explicitly; instead, asymptotic estimates of the component modal properties are employed. There are clear similarities between the basis functions used in Resound and those used in a component mode synthesis model [4]. Indeed, Resound may be viewed as a form of statistical component mode synthesis.

3. Reduction of the equations of motion

The partitioned equations of motion for a given system can be written as

$$\begin{bmatrix} \mathbf{D}_{gg} & \mathbf{D}_{gl} \\ \mathbf{D}_{lg} & \mathbf{D}_{ll} \end{bmatrix} \begin{bmatrix} \mathbf{q}_g \\ \mathbf{q}_l \end{bmatrix} = \begin{bmatrix} \mathbf{f}_g \\ \mathbf{f}_l \end{bmatrix} \quad (1)$$

where **D** is the dynamic stiffness matrix, **f** is the generalized force vector, **q** is a vector of displacements and the subscripts l and g represent the local and global degrees of freedom respectively. From the previous discussion it is apparent that there are likely to be far more local degrees of freedom than global degrees of freedom in the model. The large dimension of the local partitions typically renders a deterministic analysis of the system computationally impractical. It is therefore beneficial to reduce the local degrees of freedom from the above equation. It can be shown [6] that the perturbation to the mn'th entry of the global dynamic stiffness matrix is then given by

$$\left(\Delta \mathbf{D}_{gg}\right)_{mn} = \sum_i \alpha_i \beta_{i,mn} \quad (2)$$

where the summation i is over all local component modes in the various fuzzy subsystems and where

$$\alpha_i = \frac{\omega^4}{\omega_i^2(1+i\eta_i)-\omega^2}$$
$$\beta_{i,mn} = \left(\int_{V'} \rho \boldsymbol{\phi}_{g,m}^T \boldsymbol{\phi}_{l,i}\, dx\right)\left(\int_{V'} \rho \boldsymbol{\phi}_{g,n}^T \boldsymbol{\phi}_{l,i}\, dx\right) \quad (3)$$

The term α_i accounts for frequency effects and depends on the distribution of the local natural frequencies about the excitation frequency. The term β_i accounts for spatial effects and depends on the local and global mode shapes.

4. Asymptotic fuzzy coupling

In principle, the previous expression could be evaluated exactly for any given system. This would require the 'exact' local natural frequencies and modes shapes to be calculated in order to accurately determine α and β. There are a number of reasons why this approach is not beneficial. Firstly, there may be a significant number of local modes, which would render an exact deterministic calculation computationally impractical. Secondly, an exact deterministic calculation does not account for the effects of uncertainties in the local natural frequencies and mode shapes. The extra effort required to perform the deterministic calculation is therefore unlikely to result in an increase in the accuracy of the predictions.

One of the fundamental features of the Resound approach is that the summation in equation (2) is replaced by an integration over various regions of wavenumber space. The influence of the local modes on the global dynamic behavior can then be accounted for without having to explicitly calculate the mode shape and natural frequency associated with each local mode (resulting in a significant reduction in computational expense). Asymptotic expressions are derived for α and β as discussed in [1,6,7]. The overall approach remains computationally tractable yet captures the overall dynamic behavior of the system in a manner that is not possible using FE or SEA in isolation.

5. Acknowledgements

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6. References

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