Sound Transmission Loss of Insulating Complex Structures

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1. INTRODUCTION

Analytical and numerical modeling of the structural vibration and acoustic phenomena are essential tools in the design work of aeronautical, automotive or architectural fields. Being easier to use and faster, the analytical models are sometimes preferred to the numerical models. In spite of these qualities, the use of the analytical models is limited to the study of simple geometrical configuration; For instance, the wave approach used in the context of laterally infinite structure is very difficult to extend and implement for complex structures.

In this paper the Transfer Matrix Method (TMM) is extended to handle the transmission loss of finite stiffened and orthotropic structures lined by porous materials in a multi-layered configuration. The effects of curvature, stiffeners and heterogeneity are considered. The developed model is easily adaptable to multi-layer configurations. Moreover, the condition of fast convergence towards the solution remains a priority.

2. THE EFFECT OF THE FINITE SIZE STRUCTURE

In a recent paper [1], the radiation efficiency and the transmission index of laterally finite plane structures are evaluated by a spatial windowing method. The authors [1] show that the transmission index of the finite size structure τ_{finite} can be expressed according to the transmission index of the infinite structure $\tau_{infinite}$ corrected by a factor :

$$\tau_{finite}(\theta, \varphi) = \tau_{infinite}(\theta, \varphi)\, [\sigma\,(k_0 \sin\theta, \varphi) \cos\theta]^2\,; \qquad (1)$$

where, θ is the plane wave incidence angle, φ the propagation direction of the structural wave, k_0 the acoustic wave number and σ the non-resonant radiation efficiency. The non-resonant radiation efficiency does not depend on the properties of the structure but only on its side dimensions. The transmission index of a laterally finite multi-layered structure can be estimated using the same correct factor $[\sigma\,(k_0 \sin\theta, \varphi) \cos\theta]^2$. Thus for a given triplet $(k_0 \sin\theta, \varphi)$ the radiation efficiency must be evaluated. This evaluation necessitates a triple integrals on θ, φ and k. This process is numerically expensive which makes the method less attractive.

In the following a variant approach for calculating the non-resonant radiation efficiency is presented. This method is based on a direct semi-analytical evaluation of the radiation impedance of the structure. The radiation efficiency of a structure is defined as the ratio of the averaged sound power radiated by the structure and the sound power radiated by a piston having the same surface and same quadratic velocity as the structure. Lets consider the radiation efficiency of the finite baffled plate structure as:

$$\sigma = \frac{\Re e(Z)}{\rho_0 c_0 S}\,; \qquad (2)$$

where, ρ_0 is the air density and c_0 the sound speed in air; S is the plate aria and Z the radiation impedance given by:

$$Z = j\rho_0\omega \int_0^a\int_0^b\int_0^a\int_0^b W_n(x,y)G(x,y,x',y')W_n^*(x',y')dxdydx'dy'\,; \qquad (3)$$

where, a and b are respectively the length and the width of the piston, W_n is the displacement field of the plate and G the Green function expressed as:

$$W_n = e^{-j(k_p x\cos\varphi + k_p y \sin\varphi)}\,; \qquad (4)$$

$$G(x,y,x',y') = \frac{e^{-jk_0 R}}{2\pi R}\,; \qquad (5)$$

where, $R = \sqrt{(x-x')^2 + (y-y')^2}$ and k_p is the structural wave number.

After three different changes of variables one can write the radiation impedance as:

$$Z_{mnpq} = j\rho_0\omega \frac{ab^2}{4\pi}\int_{-1}^{1}\int_{-1}^{1}(1-u)(1-u')K(u+1,u'+1)F_n(u+1,u'+1)dudu'\,; \qquad (6)$$

where,

$$K(u,u') = \frac{e^{-j\frac{k_0 a}{2}\left[u^2+\frac{u'^2}{r^2}\right]^{1/2}}}{\left[u^2+\frac{u'^2}{r^2}\right]^{1/2}}; \tag{7}$$

and

$$F_n(u,u') = e^{-j\frac{k_p a}{2}\left[u\cos\varphi+\frac{u'}{r}\sin\varphi\right]}. \tag{8}$$

The expression of the radiation impedance (6) is now easily integrated using Gauss numerical integration scheme.

3. CURVATURE EFFECT

To model the effect of curvature the model of Donnell for shells is used [2]. In this context, the displacement field of the shell respects the assumptions of Love-Kirchhoff but takes account of the effect of curvature (radius of the cylinder R). Using these assumptions and the method of minimum potential energy the dynamic equilibrium equations of the shell are found. The nontrivial solution to this equations system leads to the dispersion relation (9), while the particular solution allows the writing of the acoustical impedance of the shell (10) :

$$k_s = \frac{m_s \omega^2}{D}\sqrt[4]{1-\frac{\omega_r^2}{\omega^2}\cos^4\varphi} \tag{9}$$

$$Z_s = j m_s \omega\left[\frac{k_s^4}{k_p^4}-\frac{k_0^4}{k_p^4}\cos^4\theta\right] \tag{10}$$

where, k_s is the structural wave number of the shell, m_s is the mass per unit area of the shell, ω_r the ring frequency of the shell, ω the circular frequency and k_p the wave number of the plate having the same side dimensions and thickness as the shell ($R\to\infty$). The relation (10) is similar to the acoustical impedance proposed by Koval [3] when the shell internal pressure and the external airflow excitation are neglected.

4. RESULTS AND CONCLUSIONS

The case of a finite (L_x=1.4m, L_y=1.1m, h=15mm) gypsum plate (E=2.5GN/m^2; ρ=690kg/m^3; ν=0.1; η=5%) is considered in order to valid the new approach. The structural averaged radiation efficiency is computed firstly by the Leppington [4] model and compared to the new approach (figure 1); the same results are obtained in both cases.

The second case presented (figure 2) show the transmission loss of an aluminum plate (L_x=1.4m; L_y=1.1m; h=1.1mm; E=70GN/m^2; ρ=2700kg/m^3; ν=0.33; η=1%). The two models are compared with the experimental data and the law mass. Note that, for the transmission loss computation by the new approach, just the radiating field are windowed.

ACKNOWLEDGEMENTS

The authors would like to thank NSERC, IRSST and Bombardier Aerospace for their financial support.

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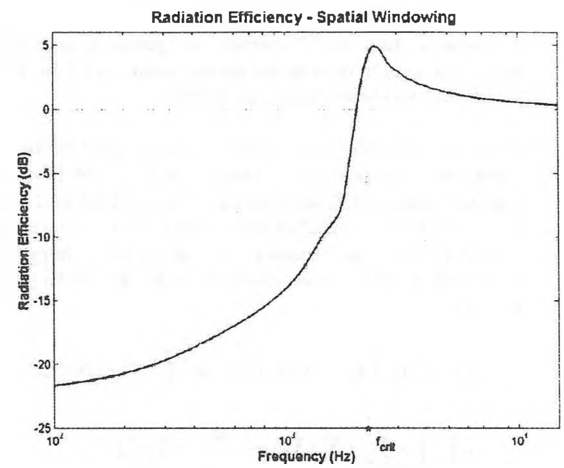

Figure 1. Radiation Efficiency for a simple gypsum plate [1] (—Leppington model, - - - Presented approach)

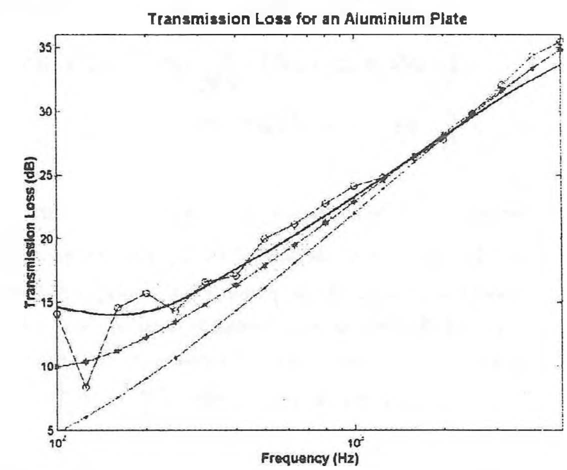

Figure 2. Transmission Loss for an aluminum plate [1]: —o— Experimental Data, — Villot and all. model, —*—New approach, —•— Infinite plate – non windowed)