The Vibro-acoustics of a Plate-Backed cavity with Non-Homogeneous Porous Materials

C. K. Amedin*, N. Atalla* and F. Sgard**

*Department of Mechanical Engineering, Université de Sherbrooke, Boul. Université, Sherbrooke, Quebec, Canada J1K 2R1

** LASH, Entpe, Rue Maurice Audin 69518 Vaulx-en-Velin, Cedex, France

1. Introduction

Various studies are performed recently to improve the acoustic performance of porous materials, which is very poor in low frequencies. Most of these studies investigate 3-D complex configurations with the finite element method. The present study presents results which indicate that use of heterogeneous material is a probable way to increase the acoustic performance. A heterogeneous material can be obtained by inclusion of elastic patches or air pockets in a homogeneous porous material. Accurate models are presented to take account of various domains (porous, fluid and elastic), and multilayer configurations are used to illustrate the advantages of a heterogeneous porous material.

2. Theoretical background

To allow a study of 3D complex configurations, finite element models are used to describe the porous, elastic and fluid domains which make up a heterogeneous multilayer.

The model associated to the porous media is based on the mixed displacement-pressure (*u*, *p*) formulation [1, 2] of Biot's poro-elasticity equations [3], which considers the solid and fluid phase of a porous material. A modified weak integral form of these equations is used to depict the boundary terms in a suitable form for the application of the coupling conditions with other media [4]. This modified form is:

$$
\begin{aligned}
&\int_{\Omega_p} \underline{\underline{\hat{\sigma}}}^S(\underline{u}) : \underline{\underline{\varepsilon}}^S(\delta \underline{u})\, d\Omega - \omega^2 \int_{\Omega_p} \tilde{\rho}\, \underline{u} \cdot \delta \underline{u}\, d\Omega \\
&+ \int_{\Omega_p} \left[\frac{\phi^2}{\omega^2 \tilde{\rho}_{22}} \underline{\nabla} p \underline{\nabla} \delta p - \frac{\phi^2}{\tilde{R}} p \delta p \right] d\Omega \\
&- \int_{\Omega_p} \frac{\phi^2 \rho_0}{\tilde{\rho}_{22}} \delta(\underline{\nabla} p \cdot \underline{u})\, d\Omega \\
&- \int_{\Omega_p} \phi \delta(p \cdot \mathrm{div} \underline{u})\, d\Omega - \int_{\partial\Omega_p} (\underline{\underline{\hat{\sigma}}}^t \cdot \underline{n}) \cdot (\delta \underline{u})\, dS \\
&- \int_{\partial\Omega_p} \phi (U_n - u_n) \delta p\, dS = 0
\end{aligned}
$$

where Ω_p is the porous domain, $\delta\Omega_p$ is the boundary surface of Ω_p, $\underline{u}$ and p are the solid phase displacement vector and the interstitial pressure of the porous-elastic medium, respectively; $\delta \underline{u}$ and δp refer to their admissible variation, respectively; $\underline{n}$ denotes the unit normal vector external to the bounding surface; $\underline{\underline{\varepsilon}}^S$ is the strain tensor of the solid phase; $\underline{\underline{\tilde{\sigma}}}^S$ and $\underline{\underline{\tilde{\sigma}}}^t$ are the *in vacuo* stress tensor and the total stress tensor of the material, respectively; U_n and u_n refer to the normal component of the solid and fluid macroscopic displacement vectors, respectively; *h* stands for the porosity of the material; ρ_0 is the air density, and $\tilde{\rho}$ and $\tilde{\rho}_{22}$ are specific mass coefficients [3]; $\tilde{R}$ refers to the bulk modulus of the air occupying a fraction ϕ of a unit volume aggregate; and ω is the angular frequency. In this weak form, the boundary terms are written in terms of the total stress tensor and the net flow. These terms are always continuous at the boundary which simplifies the coupling of porous domain with fluid or elastic domain.

The model associated to the elastic domain is based on the following weak integral form:

$$
\begin{aligned}
&\int_{\Omega_e} \left(\underline{\underline{\sigma}}^e(\underline{u}^e) : \underline{\underline{\varepsilon}}^e(\delta \underline{u}^e)\, d\Omega - \rho_s \omega^2 \underline{u}^e \cdot \delta \underline{u}^e \right) d\Omega \\
&- \int_{\partial\Omega_e} \delta \underline{u}^e \cdot (\underline{\underline{\sigma}}^e(\underline{u}^e) \cdot \underline{n})\, dS = 0
\end{aligned}
$$

where $\underline{\underline{\sigma}}^e$ and $\underline{\underline{\varepsilon}}^e$ are the structure stress and strain tensors; Ω_e, and $\delta\Omega_e$ are the structural domain and its boundary; ρ_S is the structure density; $\underline{u}^e$ is the structural displacement vector, and $\delta \underline{u}^e$ refers to an arbitrary admissible variation of $\underline{u}^e$. The volume integral of the weak form represents the sum of the work done by internal and inertial forms, and the surface integral is the virtual work done by external forces applied on the surface. This weak form is also associated to a septum (screen), if the stiffness term is neglected.

For a fluid domain, the finite element formulation is based on the following weak integral form:

$$
\begin{aligned}
&\int_{\Omega_f} \left(\frac{1}{\rho_0 \omega^2} \underline{\nabla} p \underline{\nabla} \delta p - \frac{1}{\rho_0 c_0^2} p \delta p \right) d\Omega \\
&- \frac{1}{\rho_0 \omega^2} \int_{\partial\Omega_f} \frac{\partial p}{\partial n} \delta p\, dS = 0
\end{aligned}
$$

where Ω_f and $\delta\Omega_f$ refer to the fluid domain and its boundary; p is the acoustic pressure in the fluid medium, and δp refers to an arbitrary admissible variation of *p*; ρ_0 and c_0 are the fluid medium density and the sound speed in the medium, respectively. The volume integral of the weak form represents the sum of the work done by internal and inertial forms, and the surface integral is the virtual work due to an imposed motion on the surface.

These different weak integral forms are used to build an accurate finite elements code to predict and analyze the acoustic performance of multilayers composed from elastic, porous and fluid domains. Various vibro-acoustics indicators cans be calculated for this analyze. Examples include the dissipated powers by domain, and quadratic pressure or quadratic velocity.

3. Results

Multilayer configurations are proposed here to show the acoustic performance of a heterogeneous porous material. The basic configuration consists of a plastic foam coated aluminum plate backed by an air filled cavity. The studied configurations are variant of this basic configuration, by addition of air pockets or elastic patches in the porous layer. Each configuration can be excited by a normal , oblique wave plane, or by a diffuse field, The mean quadratic pressure in the cavity is used as an acoustic indicator. The plate is in 5 mm thick, the foam is a 5.08 cm thick and the cavity is 50 cm deep. The lateral dimensions of the multilayers are 80 cm and 60 cm.

Figures 1 compares the mean quadratic pressure in the cavity, between the basic configuration and configurations where air pockets are randomly distributed in the porous layer. The configurations are excited by an oblique plane wave. It is noted that there is no significant reduction of the mean quadratic pressure in the cavity, when air pockets occupy 10 % or 20 % of the porous volume. Reduction is perceptible only at frequencies greater than 350 Hz. However, these cases are interesting because they indicate that a diminution of the weight (by insertion of air pocket) do not alter the acoustic performance.

Figures 2 illustrates the comparison between the basic configuration and others configurations where elastic patches are randomly included in the homogeneous domain, with an oblique plane wave excitation. In these cases, a significant reduction of the mean pressure is noted at low frequencies. It appears that insertion of 3 % of lead gives better acoustic efficiency that 20 % of aluminum. The result is particularly interesting since 3 % of lead is equivalent to 17 kg/m2 of mass added to the basic configuration, while 20 % of aluminum gives 27 kg/m2 of added mass. Nevertheless, it is still preliminary to say if the obtained improvement justifies the added mass.

Conclusion

The effects of insertion of air pocket or elastic patches in a homogeneous porous domain are studied in this project. The different domains are modeled by using of accurate finite elements methods. The study reveals that the addition of air pockets doesn't deteriorate the acoustic performance, but leads to a reduction of the weight of the configuration. On the other hand, an increase of the acoustic performance is obtained by insertion of elastic patches, but this comes with an increase of the configuration weight.

References

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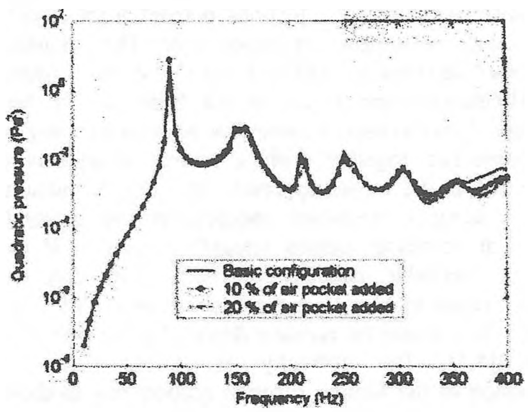

Figure 1. Effects of inclusion of air pocket s in homogeneous porous domain.

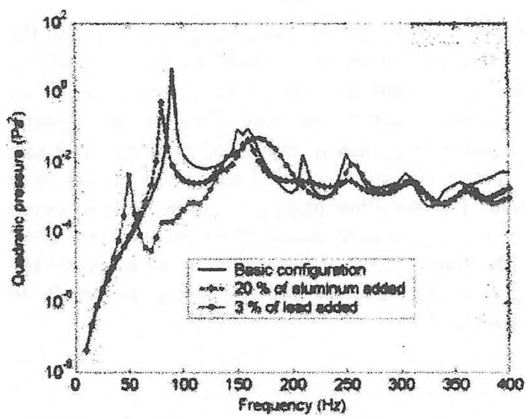

Figure 2. Effects of inclusion of elastic patches in homogeneous porous domain.