EIGENFREQUENCY EXTRACTION BASED ON BOUNDARY ELEMENT METHOD

Z. Zhang and S. T. Raveendra
Global Product Development Division, Collins & Aikman Corporation, Plymouth, Michigan 48170, USA.

1. INTRODUCTION

Acousitc eigenfrequencies are important in many applications. A widely used computational simulation technique for the analysis of acoustic problems is boundary element method. However, the use of the boundary element method for the extraction of eigenfrequencies is not as widespread since the techniques available are not very effective for the solution of problems with complex geometry and boundary conditions.

In general, two approaches are used for the extraction of acoustic eigenfrequencies by boundary elment method. In the first approach, a determinant search method (DSM) is used to solve a system of equations derived from forced response analysis integral equation [1,2]. The solution process based on DSM is inefficient and also difficult when the eigenfrequencies are closely spaced. Subsequently, the computational inefficiency is somewhat reduced by using a DSM approached together with a matrix interpolation technique [3]. In the second approach, an integral equation is derived using a frequency independent fundamental solution. The resulting domain integral is eliminated by using the particular solution of the inhomogeneous differential equation based on the approximation of the forcing function within the acoustic domain by interpolation functions [4,5]. The difficulty associated with the approximation of the forcing function renders this method not so useful in the solution of problems with complex geometry.

An alternative approach is developed here by recasting the nonlinear acoustic eigenvalue problem to a standard eigenvalue form through the linear interpolation of boundary element system matrices. That is, the system matrix at a given frequency is expressed in terms of system matrices at two closed spaced frequencies using liner interpolation. The resulting matrix equation is then solved by using readilly available standard eigenvalue extraction routines. The applicability of the technique is demonstrated by solving example problems and comparing the results to alternative solutions.

2. FORMULATION

The indirect boundary integral equation for a homogeneous acoustical cavity with rigid enclosing surface Γ is [6]:

$$0 = \int_{\Gamma} \frac{\partial^2 G}{\partial n_x \partial n_y} \mu^y d\Gamma_y$$

where G is the fundamental solution and μ is the double layer potential. The solution to the above equation can be obtained through the minimization of a functional deroved using variational approach. The resulting sytstem of equations at a frequency f can be expressed in matrix form as

$$[A]\{\mu\} = 0$$

The matrix $[A]$ can be interpolated within a suitable frequency interval in terms of matrices at the end frequencies of this interval. Suppose that the two current end frequencies are f_a and f_b ($f_a < f_b$) and the corresponding system matrices are $[A_a]$ and $[A_b]$, respectively. The linear interpolation between these two frequencies results in the following relations:

$$[A(f)] = [B] - f^* [C] \qquad (f_a \leq f \leq f_b)$$

where,

$$[B] = \frac{f_b[A_a] - f_a[A_b]}{f_b - f_a}, \quad [C] = \frac{[A_a] - [A_b]}{f_b - f_a}$$

Thus, the eigenvalue problem becomes:

$$\{[B] - f^* [C]\}\{\mu\} = \{0\}$$

where f is the eigenvalue. The eigenvalues are extracted from the above equation using QZ algorithm [7].

3. EXAMPLES

Two example problems are used to illustrate the applicability of the technique developed in the previous section. First, the eigenfrequencies of an acoustical cavity within a generic passenger car cabin are computed. The boundary element mesh of the cavity is shown in Figure 1. The eigenvalues extracted from the present approach (COMET/BEM) are compared to alternatively computed eigenvalues (COMET/FEM) [7] in Table 1. The results from both approaches are in good agreement, although one additional eigenfreuency is found in the present approach. Next, the acoustic eigenfrequencies of a simplified aircabin are computed numerically. The boundary element model for the generic aircabin is shown in Figure 2. The comparison of the eigenfrequencies obtained using two numerical methods (COMET/BEM and COMET/FEM) is shown in Table 2. Again, excellent correlation between the reults is observed.

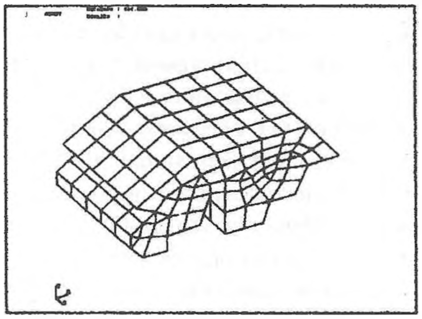

Figure 1. Boundary element model of a generic car cabin

Table 1. Comparison of the car cabin acoustic eigenfrequencies obtained using two numerical methods

COMET/FEM (Hz)	COMET/BEM (Hz)
53.042	52.199
87.167	85.335
102.036	99.931
109.144	108.041
130.818	126.994
139.690	137.373
157.198	153.349
168.645	164.820
–	172.249
182.873	180.735
189.840	186.198
190.410	194.763

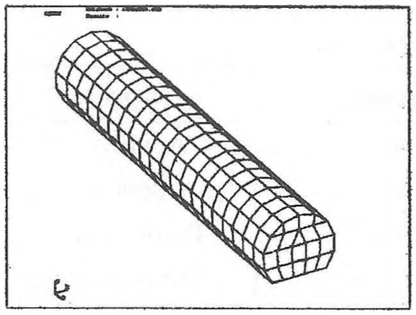

Figure 2. Boundary element model of a simplified aircabin

Table 2. Comparison of the aircabin acoustic eigenfrequencies obtained using two numerical methods

COMET/FEM (Hz)	COMET/BEM (Hz)
11.436	10.612
22.890	22.298
34.379	34.084
45.922	45.232
57.535	57.428
65.623	65.084
66.612	66.323
69.236	68.800
69.501	69.076
74.084	73.931
77.003	75.830
77.849	76.826
80.095	79.483
80.334	81.072
84.374	84.023
87.276	86.055
89.740	89.533
92.978	93.031
95.394	93.723
96.124	95.278

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