Measurement of Structural Intesntiy on Beam Structures

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INTRODUCTION

This paper provides a summary report of a recent project where a scanning laser vibrometer and specially developed analysis software were used to systematically investigate the optimal point spacing, and the suitability of measurement quality indicators, such as the residual intensity index, for the two and four point methods. Also, the sensitivity of the two-transducer method to near fields close to sources and discontinuities is also examined.

GOVERNING EQUATIONS

Although this paper focuses on beams, it is convenient to present the equations for a plate as they will be used in a companion paper (1) and represent a superset of those for a beam. It is assumed that the beam or plate is homogeneous, isotropic and that the dimension, d, in the direction of displacement is considerably smaller than the wavelength (i.e., $\lambda < 6d$). This allows thin beam/plate theory to be used and the intensity (W/m) transmitted by bending motion can be written in terms of the product of the forces (Q) and moments (M) with their corresponding normal velocity (ξ) or angular velocity (θ),

$$I_x = \langle Q_s\dot{\xi}\rangle_t + \langle M_x\dot{\theta}_x\rangle_t + \langle M_{xy}\dot{\theta}_y\rangle_t \tag{1}$$

where the subscript x and y denotes the direction and t denotes time. The first term of Eqn. 1 is the intensity due to shear forces and can be written using the bending stiffness B,

$$I_{x,sf} = -B\cdot\left\langle \frac{\partial}{\partial x}\left(\frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\xi}{\partial y^2}\right)\cdot\dot{\xi}\right\rangle_t \tag{2}$$

the second term of Eqn. 1 is the component due to bending moment and can be written as,

$$I_{x,bm} = B\cdot\left\langle\left(\frac{\partial^2\xi}{\partial x^2}+\mu\cdot\frac{\partial^2\xi}{\partial y^2}\right)\cdot\left(\frac{\partial\dot{\xi}}{\partial x}\right)\right\rangle_t \tag{3}$$

and third term of Eqn. 1 is the term for intensity due to twisting moment and can be written as,

$$I_{x,tm} = B\cdot(1-\mu)\cdot\left\langle\frac{\partial^2\xi}{\partial x\partial y}\cdot\frac{\partial\dot{\xi}}{\partial y}\right\rangle_t \tag{4}$$

For a plate, intensity is transported by all three components. While for a beam, the intensity is transported by only shear forces and bending momements. The relative magnitude of the force and moment components depends on the presence or absence of discontinuities such as sources, sinks, joints, etc. In the free field, the force and the two moment components are equal.

For a beam, the challenge is to obtain accurate estimates for the spatial derivatives of the flexural displacement in Eqns 2 and 3. Assuming free field conditions, Noiseux (2) using a finite difference approximation to provide a simplified description using only the velocity signals at two points,

$$I_x = -\frac{2\sqrt{B\cdot m'}}{\Delta}\cdot \mathrm{Im}\{G_{12}\} \tag{5}$$

where G_{12} is the cross spectrum between the velocity signals measured at two points indicated by the subscript, Δ is their spacing, and m' is the material surface density.

Similarly, the third order spatial derivatives of Eqns 2 and 3 can be estimated (3) using finite difference approximations and the measured velocity at four equally spaced co-linear measurement points. The resulting equations are,

$$I_{x,sf} = \frac{B}{\omega\cdot\Delta^3}\cdot\langle \mathrm{Im}(6G_{32}-G_{31}-G_{42}+G_{12}-G_{43})\rangle_t \tag{6}$$

$$I_{x,bm} = \frac{B}{\omega\cdot\Delta^3}\cdot\langle \mathrm{Im}(2G_{32}-G_{31}-G_{42}-G_{12}+G_{43})\rangle_t \tag{7}$$

where ω is the angular frequency. The total intensity is the sum of the two components.

MEASUREMENT SYSTEM

The measurement system consisted of a steel beam (1000x19x4.8 mm) which had one end free and the other clamped. Viscoelastic damping compound covered 400 mm of the beam at the clamped end while the free end was excited using an electrodynamic shaker coupled via an impedance head. Assuming no twisting motion of the beam and the force is applied perfectly normal to the beam then the source appears as a point force and the injected power can be accurately estimated from the force and acceleration signals from the impedance head.

A scanning laser vibrometer (Poltytec PSV300) was used to excite the beam (using a synchronized source) and to measure the resulting velocity at a series of closely spaced points along the beam from the source to the clamped end. Since the PSV 300 system measures only a single point at a time the phase relationship between the points must be obtained using the complex transfer function between the excitation signal (force from the impedance head) and the measured velocity at each point (4). Proprietary software was written to compute the structural intensity for the two and four-point methods.

SENSITIVITY OF THE METHOD TO POINT SPACING

It has been recognised that there is an optimal spacing between measurement points and that this will be a function of the wavelength. One study has suggested an operating range of $0.15\lambda<\Delta<0.2\lambda$ for the two-point method. However, a systematic investigation of the bias has not been conducted for either the two- or four point methods.

Figure 1 shows the error in intensity estimate as a function of the ratio of point spacing to wavelength for both the two and four-point methods. For both methods the choice of spacing between measurement points is critical to attaining an accurate intensity estimate, as there is a bias. A very small spacing causes an overestimation while a large spacing causes an underestimation. A very large spacing may also result in an incorrect estimate of the intensity direction. It is quite clear that the four-point method is considerably more sensitive to the spacing between points and produces very large errors outside a very small range centered about 0.35λ. For the two-point method there will be no bias when the spacing is 0.25λ.

NEARFIELD EFFECTS AND MEASURMENT METHODS

Figure 2 which shows the measured intensity on the beam as a function of the measurement position from the source indicates that for distances greater than 100 mm from the source, the far field, the moment and force components reported by the 4-point method of the intensity are equal. However closer to the source nearfield effects become important and the two components are not equal. This is the regime where the two-point method reports erroneous results as shown by the under then overestimation of the intensity. (The four-point method requires more points and a larger spacing so it is not possible to measure as close to a source).

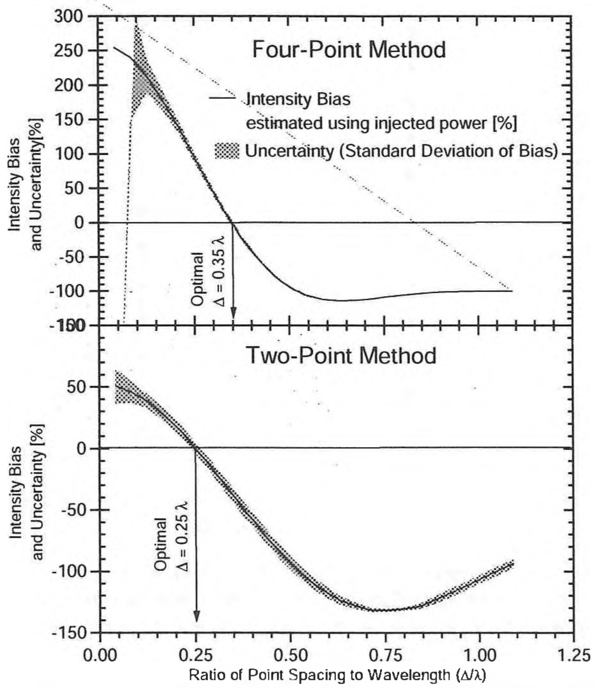

Figure 1: Bias and uncertainty in measurements as a function of the ratio of point spacing, Δ, to wavelength, λ.

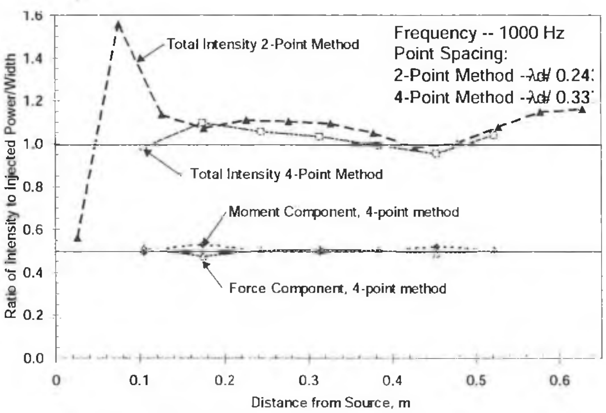

Figure 2: Ratio of intensity measured using the vibrometer and the injected intensity (power/beam width).

A MEASUREMENT FIELD QUALITY INDICATOR

For both acoustic and structural intensity measurements the ideal measurement condition is a field that consists only of a single free propagating wave. In this idealized situation there are no other sources and can be approximated by a single source in an anechoic space. For non-disipative media, the pressure/force and velocity will be in phase so the measured intensity can be written in terms of the rms velocity at the measurement point. However, when there are multiple incoherent sources there may be a very low intensity due to interference but the resulting rms velocity will be high. Accurate measurements in these situations require very precise phase information at the measurement points. The residual intensity index, R_{II}, compares the measured intensity to that predicted from the rms velocity assuming a single free propagating wave,

$$R_{II} = 10\cdot\lg\left(\frac{|I|}{\left|2\cdot k\cdot\sqrt{m' B}\cdot G_{vv}\right|}\right) \tag{8}$$

and large negative values indicate a highly reverberant field one for which even small phase mismatches may cause large random errors. While values close to zero indicate ideal conditions. Figure 3 shows the magnitude of the error for a reasonably small range in R_{II}. Since changing the ratio Δ/λ introduces a bias the range was controlled (0.23λ<Δ0.27λ) keeping the bias error typically less than ±15%. (The reason for the outlying data point is not known). The error in the measurement is not very strongly correlated R_{II}, at least for the limited R_{II} range investigated here indicating that the phase matching adequate. However, there is a slight trend to increasing uncertainty with increasing R_{II}.

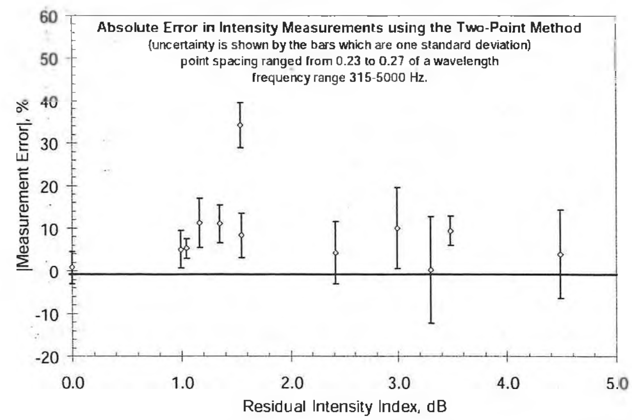

Figure 3: Error and uncertainty in the measured intensity of the two-point method as a function of the residual intensity index, R_{II}.

DISCUSSION AND CONCLUSIONS

The two-point structural intensity method provides acceptable accuracy if the measurement positions are not in the near field. In this situation the four-point method should be used. Correct selection of point spacing is critical to both methods. If an error of approximately 1dB (±30%) in the intensity can be tolerated then the point spacing should be in the following range:

Two-point 0.16λ < Δ < 0.33λ, with 0.25 λ being optimal;

Four-point 0.31λ < Δ < 0.39λ, with 0.35 λ being optimal.

The correlation between R_{II} and the measurement uncertainty was not very good. Nevertheless, it is still a useful indicator of the potential uncertainty in a measurement.

REFERENCES

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