Finite Element Modeling of Acoustical Silencers

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1. Introduction

Due to the increasing level of public awareness for noise concerns, the use of acoustical silencers is becoming more prominent. Current methods for design and prediction of performance are only reasonably accurate for specific design cases, and are unable to handle the wide variety of geometrical, environmental, and material parameters available. A numerical method, which can handle all of the various design cases and parameters, and can be implemented along with an optimization scheme is desirable.

The purpose of this paper is to provide an overview of the methods involved in creating a numerical model used to characterize an acoustical silencer. Such areas as geometry, sound absorbing material, and environmental conditions are included in the numerical model, and transmission loss results for various design cases are shown and compared to measured values obtained in physical systems.

2. Theory

The Finite Element Method (FEM) is a numerical modeling technique that can be adapted for use with acoustical problems. Starting with the acoustic wave equation, the acoustic FEM is given as follows [1]:

$$\Big(\overbrace{[K_m]}^{\text{Stiffness}} + j\overbrace{k}^{\text{Wave Number}}\big(\underbrace{R}_{\text{Resistivity}}\Omega/\rho \underbrace{c}_{\text{Speed of Sound}}\big)\overbrace{[M_m]}^{\text{Mass}} - k^2 \underbrace{K_s}_{\text{Structural Factor}} \underbrace{\Omega}_{\text{Porosity}} [M_m]\Big)\overbrace{\{p_m\}}^{\text{Pressure}} = (j\overbrace{\rho}^{\text{Density}}\underbrace{\omega}_{\text{Frequency}} + R)\overbrace{\{Q_m\}}^{\text{Velocity}} \qquad (1)$$

This formulation is analogous to a multi-degree of freedom vibration problem. In this case, the pressure vector is the desired solution. In order to model an acoustical silencer, the geometry is divided up into a meshed grid of acoustical elements (as illustrated in Figure 1).

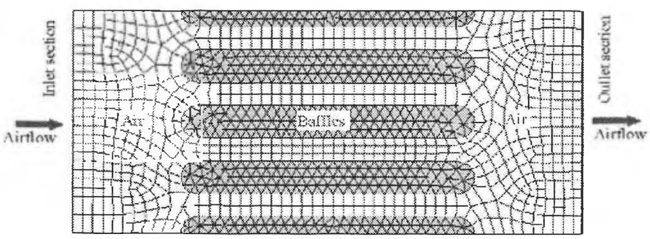

Figure 1. FEM Mesh of Parallel Baffle Silencer

Each element is interconnected with its immediate neighbor, forming a global matrix over the entire geometry.

The next step in the numerical model is to define the fluid and absorptive material properties. Such fluid properties as the speed of sound, the density, and the temperature can be altered. The absorptive material properties which can be included in the model are the flow resistivity, the porosity and the structural factor [2]. The values for these properties can either be known before hand to predict the response of a known system, or can be altered to aid in design and optimization.

The final step in the model is to apply the boundary conditions. In order to excite the model, the elements at the inlet section are given a unit particle velocity. The elements at the exit section are given the characteristic impedance ($z = \rho c$) to mimic an open section (preventing waves from reflecting once they have left the silencing element).

Once all of the elements have been formed and assembled, Eqn. (1) is solved for the pressure vector at each of the element nodes. The three pressure values of interest (p_1, p_2 and p_3) are interpolated from the resulting pressure vector and are then used in the 3-point method for calculating Transmission Loss [3] (*TL*, defined as the ratio of sound intensity incident to sound intensity transmitted). The 3-point method measures the sound at two points upstream from the silencer and one point downstream (as shown in Fig. 2).

Figure 2. Measurement Locations for 3-Point *TL* Formulation

The resulting equation can then be used to solve for the *TL* in the numerical silencer model.

$$TL = 20\log_{10}\left|\frac{p_1 - p_2 e^{ikx_{12}}}{p_3 - p_3 e^{i2kx_{12}}}\right| \qquad (2)$$

3. Discussion of Results

Verification of the results was completed by comparing the numerical model results to those obtained using known formulations of simple reactive silencer systems, and more complex physical models of parallel baffle silencers.

One of the first verifications performed, involved the modeling of a simple expansion chamber silencer. A physical model was constructed and it's *TL* was measured. Also, based on the dimensions, its *TL* was predicted using the known formula for an expansion chamber [4]. Both of these results were compared to those obtained by a numerical model of the same dimensions. Figure 3 shows the dimensions of the silencer in question while Fig. 4 shows measured, calculated, and numerical results.

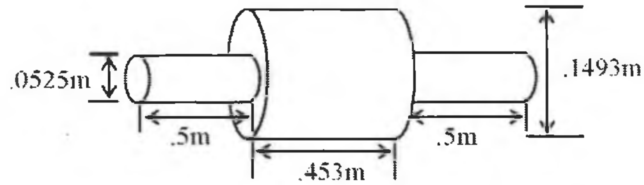

Figure 3. Expansion Chamber Silencer Dimensions

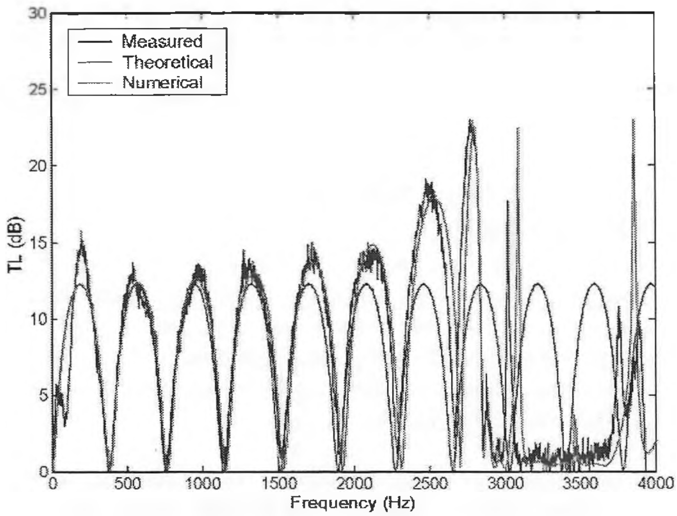

Figure 4. Expansion Chamber *TL* Results

Note that all three curves follow each other until the critical frequency at which the plane wave propagation assumption is no longer valid (approx. 2000Hz). After this point, the theoretical prediction is no longer valid, and can be ignored. The measured and numerical curves, however, follow each other very well over the entire frequency spectrum.

Another verification of the FEM involved the modeling of a scale parallel baffle silencer. The numerical results were compared to those measured from the scale model shown in Fig. 5 which consisted of a source end with a straightening section (for plane wave propagation), a test section with variable parallel baffle configurations and numerous microphone locations, and a termination section with an anechoic termination to prevent reflected waves from returning after the sound has left the test section.

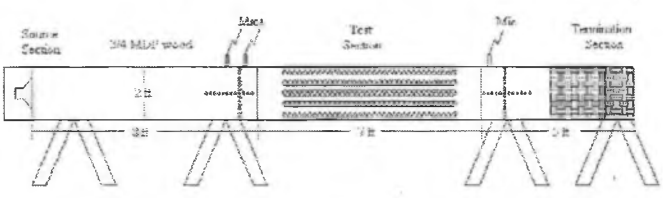

Figure 5. Physical Model of Acoustical Duct Silencer System.

The test section contained 3 baffles of 50mm thick Kaowool Ceramic Fiber with a flow resistivity of 106000mks rayls/m, a porosity of 0.799 and a structural factor of 2.0. Figure 6 shows the results obtained from the numerical and physical models, along with the difference between the two.

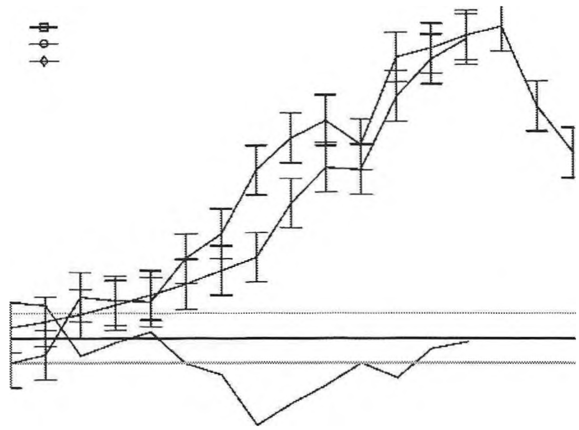

Figure 6. TL results for 3 baffles of Kaowool Ceramic Fiber

Note that for most of the frequency range tested, the difference between the two curves is within ±5dB and the error bars (based on statistical testing) almost always overlap

4. Summary and Conclusions

The results gathered thus far indicate excellent results for purely reactive acoustical silencers, and good results for absorptive silencers. Future work would include a more complicated model for sound absorbing material, and the inclusion of flow. Ultimately, external calculations will be required (either by Boundary Element Methods or Infinite Element Methods) along with vibro-acoustic coupling to calculate the Insertion Loss.

Acknowledgements

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