Vibro-acoustic behaviour of multi-layer orthotropic panels

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1. INTRODUCTION

Multi-layer composite panels such as sandwich and honeycomb are widely used in the aerospace and aeronautic industries. A good understanding of their behaviour is necessary to predict their dynamic behaviour under acoustic excitation. A general eigenvalue approach based on wave theory is presented to compute the dispersion curves of such panels. Using these curves, the radiation efficiency, the modal density and the group velocity are computed and used within SEA framework to predict the transmission loss of these panels. Numerical results are presented to validate the proposed approach.

2. NUMERICAL MODEL

The literature [4], [5] proposes various methods on the dynamic behaviour of multi-layer composite structures. It is shown [5] that the analytical resolution of the general problem is not possible unless simplifying assumptions are made which narrows the field of applicability of the suggested models. In order to circumvent these disadvantages, a general numerical method is used to establish and solve the dispersion relation of these structures.

Using a general Mindlin model [1], [2] with transverse shear and the strain - deformations relations, the governing equations of the composites are written in terms of a general hybrid vector made up of the generalized efforts and displacements of the panel :

$$\langle e \rangle = \langle w, \psi_x, \psi_y, M_x, M_y, M_{xy}, Q_x, Q_y \rangle; \qquad (1)$$

with, w the displacement along z axis; ψ_x , ψ_y the rotations around y and x axis respectively; M_x, M_y, M_{xy} the bending moments and Q_x and Q_y the shear efforts. In this context, the system of equilibrium equations efforts - displacements of the panel can be written in the form:

$$[A_0]\langle e \rangle = [A_1]\left\langle \frac{\partial e}{\partial x} \right\rangle + [A_2]\left\langle \frac{\partial e}{\partial y} \right\rangle; \qquad (2)$$

where: $[A_0]$, $[A_1]$, $[A_2]$ are matrices of dimension 8×8 containing the coefficients of the equilibrium efforts - displacements equations system of the panel.

The solution of the system is written in the form:

$$\langle e \rangle = \{e\}\, e^{j\omega t \;\; jk_x x \;\; jk_y y}; \qquad (3)$$

with, k_x and k_y the components of the structural wave number k_p of the panel and ω the pulsation of vibration. The real values of k_p correspond to the propagating waves while the complex values to the evanescent waves. In this study our interest is limited to the propagating waves.

Relation (2) becomes following the use of relation (3) and algebraic simplifications:

$$[A_0]\{e\} = k_p\,[B]\{e\}; \qquad (4)$$

with:

$$[B] = -j(\cos\varphi\,[A_1] + \sin\varphi\,[A_2]); \qquad (5)$$

where, one used:

$$k_x = k_p \cos\varphi; \quad k_y = k_p \sin\varphi \qquad (6)$$

φ being the direction of wave propagation in the panel.

The resolution of the system (4) leads to a generalized eigenvalues problem, where k_p represents the eigenvalues vector and $\{e\}$ the matrix of the corresponding eigenvectors (columns). The eigenvalues vector contains the bending and evanescent wave numbers (conjugate complex solutions). The bending wave number of the panel corresponds to the mathematically real eigenvalue of the problem (with an infinitely small imaginary component).

3. GROUP VELOCITY AND MODAL DENSITY

In a structure, the waves having the same phase velocity, transport the energy of vibrations with a velocity called group velocity. It is expressed in the following general way :

$$c_g(\omega, \varphi) = \frac{d\omega}{dk}. \qquad (7)$$

Numerically, in a first approximation, the group velocity is obtained for a given heading φ using a finite difference scheme as the ratio between the infinitesimal variation of the excitation frequency and the corresponding variation of the structural wave number.

Next, the modal density is obtained by integrating its angular distribution:

$$n(f) = \int_0^{\pi/2} \frac{2L_x L_y}{\pi} \frac{k_p}{|c_g|} d\varphi \; ; \qquad (8)$$

with, L_x and L_y the side dimensions of the panel and f the frequency of excitation.

4. RADIATION EFFICIENCY

The radiation efficiency of the panel $\sigma^*(f)$ is computed using Leppington model [3] with the structural wave number, obtained by the resolution of the generalized eigenvalues problem. In order to take into account the total vibration energy of the resonant modes in a frequency band, the radiation efficiency is averaged in the wave number space by the angular distribution of the modal density $n^*(f,\varphi)$. One makes here the assumption of energy equipartition between the resonant modes in a frequency band. The radiation efficiency of the multi-layer composite panel is written as:

$$\sigma(f) = \frac{\int_{k_{min}}^{k_{max}} \int_0^{\pi/2} \sigma^*(f) n^*(f,\varphi) k dk d\varphi}{\int_{k_{min}}^{k_{max}} \int_0^{\pi/2} n^*(f,\varphi) k dk d\varphi} \; ; \qquad (9)$$

where, k_{min} and k_{max} are the wave number field boundary corresponding to the studied frequency band.

5. VALIDATION OF THE MODEL

Like first validation of the model, one considers a thick orthotropic panel (one layer) having the same physical properties in the three directions (isotropic thick panel). The results of the numerical model for the composite panels are compared with the results of the analytical model for isotropic thick panels. The case of an aluminium panel ($E=7.2*10^{10}$ Pa; $\rho=2780$ kg/m^3; $\nu=0.33$; $\eta=0.007$) of thickness h=10cm and side dimensions $L_x = 4.6$ m and $L_y = 2.3$ m is considered to validate the new approach. The modal density of the panel is calculated (figure 1) by the analytical model (—o—) and compared with numerical results (—*—). One observe that at low frequencies the effect of bending dominates while at high frequencies shearing become significant. The asymptotic tendencies in bending (—•—) and shearing (— —) are also shown for the same panel.

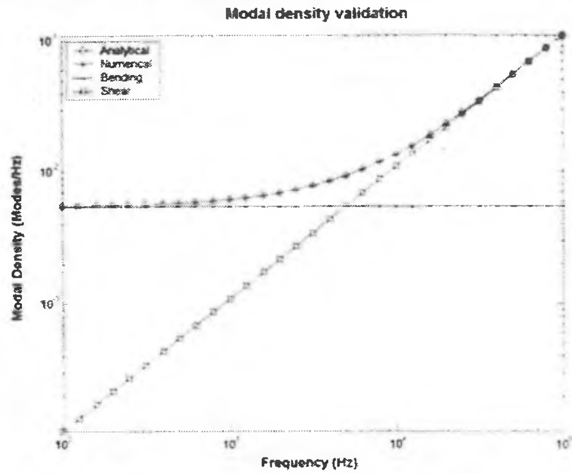

Figure 1. Modal density of the panel

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