A fully hierarchical finite elements code for solving coupled elasto-poro-acoustic problems

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1. INTRODUCTION

The design of multi-layer structures is of significant importance in several industries including automotive, acrospace and buildings. A typical application concerns a foam layer sandwiched between an elastic structure and a limp impervious layer classically known as a septum and coupled to an acoustic cavity. At low frequencies, where the modal behavior of the system is important, finite elements are normally used. Though accurate, the use of linear elements in dynamic problems leads to an important number of unknowns and thus requires important computational resources. This paper presents the performance of hierarchical elements compared to linear elements for dynamic problems. For the sake of simplicity, the performance is demonstrated for a plate backed by a rigid cavity.

2. THEORY

2.1 Hierarchical elements

The p-extension of the finite elements method adds high order polynomial shape functions for the interpolation of the field of variables on the reference elements. In addition to the node modes used by linear elements, hierarchical elements are built with side modes, face modes and also internal modes in the case of tri-dimensional elements. Some are shown in figure 1.

Because the shape functions themselves resemble the vibration modes of the structure, a quick convergence is obtained for the dynamic behavior [1].

2.2 Coupling between different media

At a specific frequency, two different media generally do not have the same wavelength. Consequently, the level of discretization required is not the same. With linear elements, one of the two domains is over-distcretized if incompatible meshes are not used.

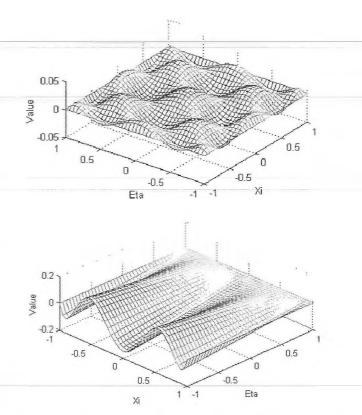


Figure 1: Side mode and face mode of order 6 on a reference quadrilateral elements.

This problem is avoided by the used of hierarchical elements. The order of approximation is not necessarily the same for the two elements in contact, or for the two phases of one elements as in the case of porous elements arising from the mixed (u,p) formulation.

2.3 Transverse shear locking

The shear locking phenomena is a well know problem for some elements built with variationnal formulation involving only displacement. With hierarchical elements, the approximation space is rich enough to avoid this problem. This does considerably facilitates the development of plate, solid and porous elements having both the solid and the fluid phase [2].

3. RESULTS

The case considered here is a plate backed by a rigid cavity. The dimensions of the plate are $0.42 \,\mathrm{m} \times 0.48 \,\mathrm{m}$. The thickness of the plate is 3 mm and the thickness of the cavity is $0.35 \,\mathrm{m}$. The properties for the aluminium plate are, $E \, (\mathrm{N/m^2}) = 70 \mathrm{x} 10^9, \ \nu = 0.33, \ \rho \, (\mathrm{kg/m^3}) = 2700, \ \eta = 0.015$ and the properties of the air are $c_0 \, (\mathrm{m/s}) = 342.2, \ \rho_0 \, (\mathrm{kg/m^3}) = 1.213$ and $\eta = 0.01$.

The excitation is a point force located at a quarter of the plate in each direction.

The performance of the hierarchical code are compared to those of the commercial code MNS/Nova®. The mesh of the plate in Nova® is 20 x24 elements and the cavity contains 7 elements in the thickness. The computation time for the hierarchical code was 10:15 min while Nova® took 59:15 min. The order of appromixation per unit length in the hierarchical code is 33 for the plate and 12.5 for the cavity. The hierarchic shape function set used is exactly same as in MSC/Probe®.

Figure 2 compares the results of the two code and also shows that the hierarchical code has converged while Nova® has not yet converged for the plate. Making Nova® converge is a hard task because the number of degress of freedom grows as the square of the number of elements on each side of the plate.

4. DISCUSSION

Here we considered only the simple case of a plate backed by a cavity. Similar performance of the hierarchical elements have been obtained for other structure comprising porous media.

The hierarchical code developed will replace the low frequency module of MNS/Nova®.

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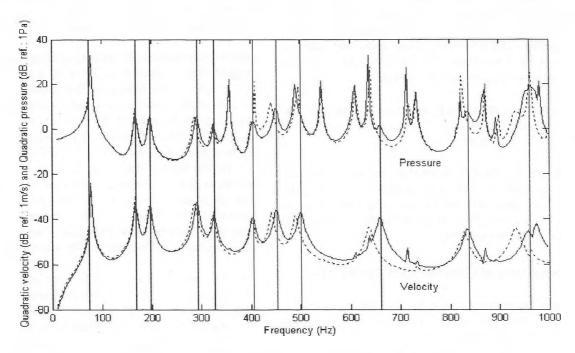


Figure 2: Comparison of the results given by the hierarchical code to those of MNS/Nova®. The dotted curve is the one of Nova® and the plain curve are the results of the hierarchical code. The vertical lines are the analytical results for the vibration modes of the plate.