BAYESIAN APPROACHES TO GEOACOUSTIC INVERSION

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1. INTRODUCTION

Great effort has been applied to estimating seabed geoacoustic properties using measured ocean acoustic fields. This amounts to an inverse problem: determine a model **m** given observed data **d**, where the model $\mathbf{m}=\{m_i, i=1,M\}$ represents the unknown geoacoustic and geometric parameters. In this paper two inversion algorithms are applied to ocean acoustic data. Adaptive simplex simulated annealing (ASSA) [1] is an optimization inversion algorithm that determines the model **m** that minimizes the objective function. While ASSA is an efficient nonlinear inversion algorithm for determining model parameter estimates, it does not provide rigorous parameter uncertainty estimates. A Bayesian sampling algorithm, the fast Gibbs sampler (FGS) [2], is also applied here. Through Bayesian inference, parameter estimates, parameter uncertainty estimates, as well as other information about the problem, can be determined.

To assess the abilities of ASSA and the FGS when applied to data from range-dependent environments, the algorithms were applied to synthetic benchmark data generated for the 2001 Inversion Techniques Workshop [3]. Some results using Test Case 1 (TC1) data for a shallow-water down-slope environment are presented here (see also [4] and [5]). An under-parameterized approach was applied to determine the optimal model parameterization for the environment.

2. BAYESIAN APPROACH

For the Bayesian approach to inverse problems [2], **d** and **m** are considered random variables. Bayes' rule states that the posterior probability density (PPD) $P(\mathbf{m}|\mathbf{d})$ is proportional to the likelihood function $L(\mathbf{d}|\mathbf{m})$ multiplied by the prior probability distribution of **m**, $P(\mathbf{m})$. The PPD embodies the general Bayesian solution to the inverse problem. Due to the PPD's multi-dimensionality, its properties can only be assessed indirectly, using, for example, the marginal probability densities, the posterior mean model, and the model covariance matrix. In addition, highest posterior density (HPD) intervals can be used to quantify parameter uncertainties. The smallest interval of each marginal density containing α% of the distribution's area defines the α% HPD. Also, the model that maximizes the PPD, the maximum *a posteriori* (MAP) solution, provides alternative parameter estimates. Under certain conditions, a sampler which samples a Gibbs distribution (a Gibbs sampler (GS)) can be used to estimate the PPD properties.

3. INVERSION METHODS

ASSA, one of the inversion methods used here, is a hybrid algorithm that combines the global and local inversion methods of simulated annealing and downhill simplex to search the parameter space for the optimal model. Each proposed model **m′** is assessed by evaluating the mismatch E between the measured fields **d** and modeled fields **d**(**m′**). The MAP is located when an objective function related to the PPD is used [5]. For a more complete solution, the FGS, which combines a GS with features that increase its efficiency, can be applied to determine the PPD properties.

The appropriate model parameterization is usually unknown in geoacoustic inverse problems. An under-parameterized approach is applied here to determine the appropriate number of sediment layers needed to represent the seabed. This approach includes repeatedly applying the inversion algorithm and increasing the number of layers each time. The solution model that minimizes both mismatch and structure will have the optimal parmeterization.

4. RESULTS

The results of applying the under-parameterized approach to TC1 using ASSA are shown in Figure 1. In Figure 1a, the mismatch E decreases significantly between L=1 and L=3, and then plateaus. Therefore, the model should include at least 3 sediment layers. As a measure of structure, the l_1 norm of variation between like parameters was calculated. In Figure 1b–d the variations of the sediment compressional

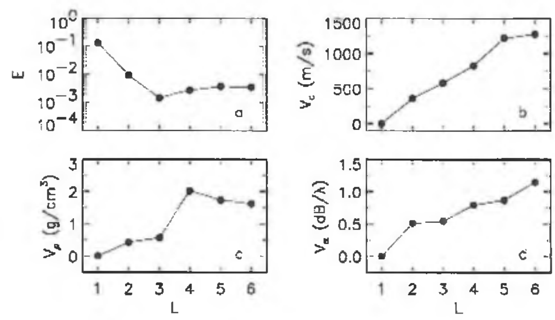

Figure 1. Under-parameterized approach to determining the appropriate number of sediment layers L using (a) the mismatch E and (b)–(d) the l_1 norm of variation for compressional speed V_c, density V_ρ, and attenuation V_α.

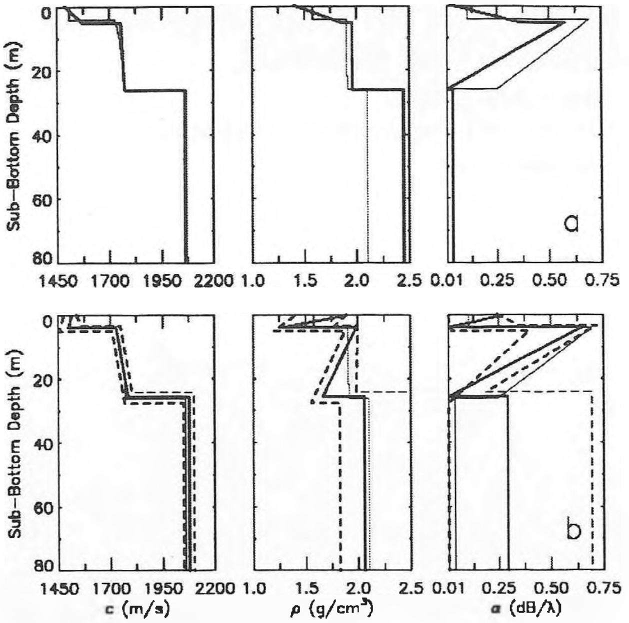

Figure 2. MAP profiles of compressional speed c, density ρ, and attenuation α using (a) ASSA and (b) the FGS. The thin line represents the true model and the thicker line represents the MAP estimates. The dashed lines represent the 95% HPD interval bounds.

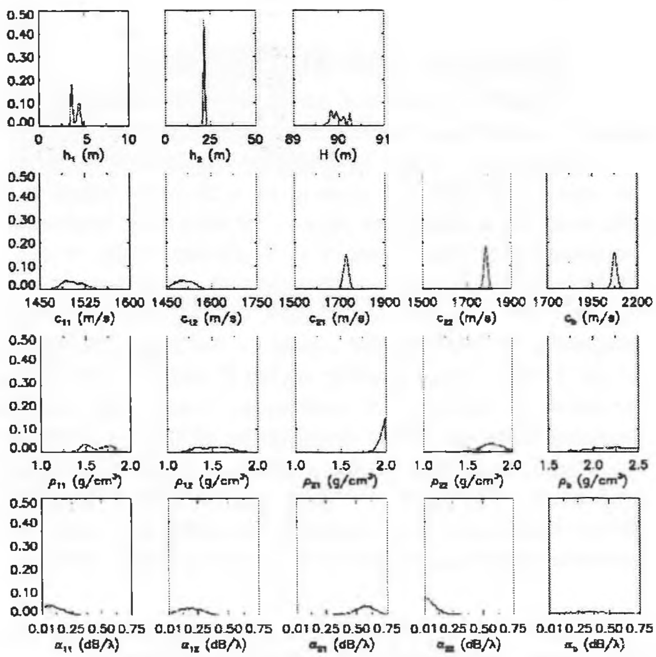

Figure 3. Estimated marginal probability densities for all parameters: layer thickness h, water depth at the source H, c, ρ, and α. For parameters x_i and y_{ij}, i represents layer (1, 2, or b (basement)) and j represents the top (1) or bottom (2) of the layer. The abscissa limits represent the bounds used in the inversion.

speed V_c, density V_ρ, and attenuation V_α typically increase with L. The model parameterization that minimizes the structure and the mismatch is, therefore, the L=3 parameterization.

Figure 2a shows the L=3 ASSA MAP estimate through parameter profiles. The true model is included for comparison. The compressional speed profile, including layer thickness, approximates the true profile extremely well. The density and attenuation profiles are also very well determined.

The estimated TC1 1-D marginal probability densities generated using the FGS for a three-layer model are shown in Figure 3. Most distributions are unimodal and symmetric. The layer thickness and compressional speed parameters have generally narrow distributions and are, therefore, well determined parameters. Density and attenuation parameters are not as well determined.

Figure 2b shows the parameter profiles for the FGS MAP estimate and the true model. The FGS MAP solution is a very good estimate of the true solution. Also included in this figure are the schematic representations of the 95% HPD intervals used to quantify the parameter uncertainties.

5. CONCLUSIONS

ASSA and the FGS were successfully applied to the range-dependent benchmark data. The appropriate number of sediment layers and good parameter estimates were determined using the under-parameterized approach and ASSA. While not as efficient as ASSA, the FGS provides uncertainty bounds which are crucial for assessing the quality of the final estimate.

6. REFERENCES

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