ELIMINATING DIVISION OPERATION IN NLMS ALGORITHM

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1. INTRODUCTION

The normalized least mean square (NLMS) adaptation algorithm is widely used in acoustic and network echo cancellation, noise cancellation, channel equalization, system identification, and so on. In each iteration of a conventional NLMS implementation, a division operation is required to update a variable called the step size. Since a division consumes much more real-time than a multiplication or addition does in a typical digital signal processor (DSP), a significant portion of the precious processing power is spent on the division operations if there is a large number of them present in the algorithm. Furthermore, a conventional NLMS algorithm does not respond to sudden increases in input level promptly enough in order for an echo canceller to meet today's stringent requirements, such as [1].

This paper proposes a patent-pending alternative[†] where the division operation is avoided so that the amount of computation for NLMS is reduced. In addition, the proposed approach can accelerate the algorithm's response to sudden increases in input level when properly implemented.

2. BACKGROUND

Beyond the scope of this paper, details about the NLMS algorithm can be found in [2]. This paper only deals with the calculation of the step size therein, which is denoted by $\mu(n)$ at sampling interval n. The conventional NLMS finds $\mu(n)$ by using

$$\mu(n) = \beta / \langle x^2(n) \rangle \tag{1}$$

† The author was with Nortel Networks when the present approach was conceived. Nortel Networks retains ownership of intellectual property rights relating to this article and its subject matter.

where β is a positive constant, $x(n)$ is the input sample at n, and <·> is the operator for a weighted time average over a certain number of past samples of the argument. Typically, $\langle x^2(n) \rangle$ is an estimate of the energy in $x(k)$ ($k = n, n\text{-}1, n\text{-}2, \ldots$) over a certain number of most recent samples.

Equation (1) indicates that a division is needed in each sampling interval n in order to calculate $\mu(n)$. It is well-known that division operations are quite expensive, in terms of real-time usage, on most commercial DSPs. For example, it takes only one instruction cycle for Texas Instruments' C54x, a typical commercial DSP family, to do a multiplication, while it takes at least 34 instruction cycles for the same processor to do a basic one-quadrant, 32-bit by 16-bit, division [3].

Thus, minimizing the number of division operations can significantly improve the efficiency of an algorithm.

3. THE PROPOSED APPROACH

The concept behind finding a quotient without performing a division is, in each sampling interval, to compare the numerator with the product between the denominator and a quotient estimate, then to adjust the latter accordingly. The tracking error can be negligibly small if the true quotient varies slowly over time, as is the case with the NLMS algorithm; the numerator in Eq. (1) is a constant and the denominator, being a time-average, is slowly time-varying.

In each sampling interval n, the proposed approach starts with $\hat{\mu}(n-1)$, an estimate of the step size used by the last sampling interval, compares β with the product of $\hat{\mu}(n-1)$ and $x^2(n)$, and updates $\hat{\mu}(n-1)$ accordingly to arrive at $\hat{\mu}(n)$, step size estimate to be used by the current sampling interval. Being a first-order closed loop feedback system, the approach is illustrated in Figure 1.

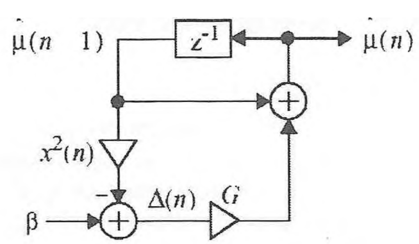

Figure 1. Flow diagram of the proposed approach

The equivalent analytical form is given by

$$\begin{aligned} \hat{\mu}(n) &= \hat{\mu}(n-1) + G\Delta(n) \\ \Delta(n) &= \beta - \hat{\mu}(n-1)\,x^2(n) \end{aligned} \quad (1)$$

where $\Delta(n)$ in Eq. (1) is the result of the comparison, and G is a positive factor that controls the rate of the adjustment.

To show that Eq. (1) gives an approximation to Eq. (1), take expectations of Eq. (1) while considering the fact that G is small so that $\hat{\mu}(n-1)$ changes slowly. The result is

$$E[\hat{\mu}(n)] = \{1 - GE[x^2(n)]\}E[\hat{\mu}(n-1)] + G\beta \quad (2)$$

Since

$$GE[x^2(n)] \ll 1 \quad (3)$$

holds, Eq. (2) converges so that $E[\hat{\mu}(n-1)] = E[\hat{\mu}(n)]$. Equation (2) can then be solved as

$$E[\hat{\mu}(n)] = \beta / E[x^2(n)] \quad (4)$$

which, under the assumption that $x(n)$ is ergodic, is equivalent to Eq. (1).

Note that $\langle x^2(n)\rangle$, as needed in Eq. (1), is not calculated explicitly here. Instead of a time-averaged version of it, only a single sample $x^2(n)$, with a much larger fluctuation, is used in each iteration. In fact since Eq. (3) holds, $\hat{\mu}(n)$ fluctuates much less than $x^2(n)$ does; therefore, as an integration of the differences $\Delta(n)$ over time, it reflects the impact of $\langle x^2(n)\rangle$ implicitly. This means that the proposed approach saves computation not only by eliminating the division operation, but also by not calculating the time-average $\langle x^2(n)\rangle$.

In practice, the real-time it takes for a typical DSP, such as the C54x [3], to perform such a "pseudo-division" as Eq. (1) is only about one fifth of what it takes to do a real division.

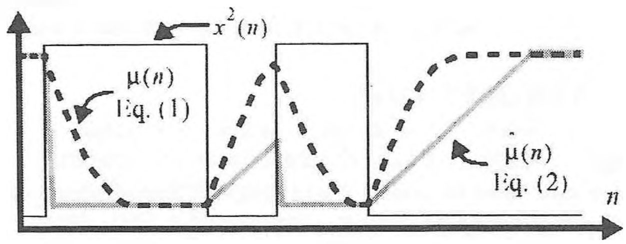

Figure 2. Behaviors of conventional, by Eq. (1), and proposed, by Eq. (1), approaches

Figure 2 illustrates that the proposed approach, given by Eq. (1), responds to sudden increases in input signal level much faster than the conventional approach, by Eq. (1), does. There are test cases required by [1] that incorporate such fluctuations, which can easily cause the conventional NLMS to diverge momentarily because of its large response time. On the contrary, an NLMS algorithm featuring the proposed approach has been proven to survive such fluctuations well. Figure 2 also shows that the proposed approach has a longer ramp-up time when the input signal level drops. This is usually not considered an issue, because it only slows down the convergence when the signal level has dropped, and never causes any concerns for divergence.

1. SUMMARY

A simple and easy to implement way of avoiding the division operation in the widely used NLMS algorithm has been studied. In addition to simplifying the implementation, the proposed approach responds to the input signal dynamics in a manner in favour of avoiding potential system divergence.

The concept in the proposed approach can be applied not only to the NLMS but also to other algorithms where a division is needed and the quotient to be estimated does not change quickly over time.

REFERENCES

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[2] Simon Haykin, *Adaptive Filter Theory*, 3rd Edition, Prentice Hall, 1996.

[3] Texas Instruments, *TMS320C54x DSP Reference Set Volume 2: Mnemonic Instruction Set*. Literature Number: SPRU172C. March 2001.