THE AGGREGATE BEAMFORMER

David I. Havelock
National Research Council, 1200 Montreal Rd., Ottawa, ON, K1A 0R6, david.havelock@nrc.ca

1. INTRODUCTION

Beamforming is used to achieve directional response using a spatially distributed receiver. Antennae have been used for analog electromagnetic beamforming in radio, radar, and microwave communication. Early digital beamformers (such as DIMUS) were developed for the detection of submarines. More advance techniques such as adaptive and synthetic aperture beamforming have now become textbook material.

These conventional beamforming techniques rely on simultaneous (or, at least, systematic) sampling of the individual array elements (channels). Here, we discuss a method of beamforming which collects each sample from a randomly selected array element. The samples collected from all channels this way are then treated together to recover the desired signal; hence the term 'aggregate beamformer'.

Whereas the conventional beamformer cancels the off-beam signals by destructive interference, the aggregate beamformer converts these undesired signals into white noise. The approach gives the same beam response pattern as conventional beamforming but exhibits an additional residual noise. Since the overall sampling rate of the aggregate beamformer exceeds the Nyquist rate for the desired signal, low-pass filtering and decimation is used to reduce the noise. The noise level can be reduced to any desirable level by adjusting the overall sampling rate.

The principle advantages of the aggregate beamformer are its ability to function without stringent anti-alias filters, it scales so that arrays with many sensor elements (hundreds or thousands) can be used without a significant increase in complexity or other component count, it provides improved beam steering resolution, it can be designed using the same principles as conventional beamformers, it has no arithmetic operations (except for the final decimation filter), and the reduced hardware requirements makes solid state integration easier.

2. CONVENTIONAL BEAMFORMING

For simplicity, we compare the aggregate beamformer here only with the conventional delay and sum beamforming (DS), although more sophisticate beamformers are readily implemented. The DS beamformer applies a delay to the signal received at each element and computes the sum of the delayed samples. The beamformer B_{ds} is

$$B_{ds}(n) = \sum_i X_i(n - d_i) \qquad (1)$$

where i is the sensor index, W_i is a channel weight, $X_i(n) = X_0(n-t_i)$ is the sampled signal. $X_0(n)$ is the source signal as observed at the array centre, t_i is the signal propagation time difference between the array centre and sensor i, and d_i is the (integer) sample delay (approximating t_i).

The sample delay quantization reduces the beamformer performance (directivity) and limits the number of distinct beams that can be formed (steering resolution). Typically, the sampling rate is increased or a digital interpolation filter is used to minimize these problems.

Numerous channels may be multiplexed to a high speed analog to digital converter (ADC) but a separate analog anti-alias filter is needed for each channel (unless separate delta-sigma ADC are used on each channel.)

3. AGGREGATE BEAMFORMING - SIMPLE BROADSIDE CASE

Consider a linear array receiving a far-field signal arriving perpendicular to the array (broadside). The wavefront arrives at all sensors at the same time so the DS beamformer output signal is simply,

$$B_{ds}(n) = (1/M)\sum_{i=0}^{M-1} X_i(n). \qquad (2)$$

If there is no noise or interference then the X_i are all identical and so one could equally well choose any value for i and define $B(n) = X_i(n)$ as the output! – however, this would not remove undesired signals arriving from other (off-broadside) directions.

Notice, however, that if the sensor index i was changed randomly so that for sample n we choose $i = s(n)$ where $s(n)$ is a random number distributed uniformly over the set of sensor indices, then we get

$$B_{abf}(n) = X_{s(n)}(n) \qquad (3)$$

which preserves a signal arriving from broadside but introduces random sampling delays that corrupt signals arriving from other directions. We call this the broadside 'aggregate beamformer' (ABF). It is easily implemented in a multiplexed ADC configuration by applying random addresses to the multiplexer.

The corrupted output (for off-broadside signals) have reduced amplitude and added noise. It turns out that the noise is white Gaussian noise and the amplitude is *exactly* the same as that which the DS beamformer would produce. For a signal arriving broadside (on-beam) the total sampling rate of the ABF need only be the Nyquist rate of the signal: this is $1/M$ the total sampling rate of the conventional DS beamformer.

The residual noise of the aggregate beamformer is reduced by increasing the sampling rate by some 'over-sampling factor' beyond the Nyquist rate. Since the residual noise is uniformly distributed over the entire output bandwidth, as in Fig 1, a lowpass filter (followed by decimation) will substantially reduce the noise power without affecting the signal. If the over-sampling factor is M then the residual noise for an off-beam signal will be.

$$\|N_{\text{res}}\| \; \frac{1}{M}(\|X_0\| + \|B_{\text{ds}}\|) \qquad (4)$$

A signal arriving broadside (on-beam) will have no residual noise.

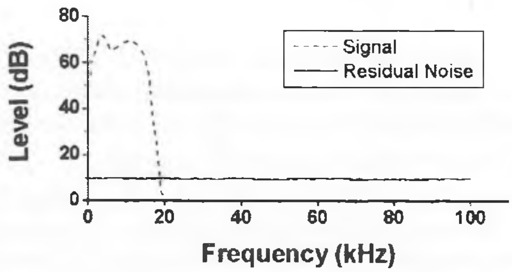

Fig. 1. Spectrum of signal and residual noise. Lowpass filtering reduces the residual noise power

THE GENERAL CASE

The aggregate beamformer can be steered to any beam direction using sample delays d^{θ}_i computed as for the conventional beamformer for a specific direction θ. Since the aggregate beamformer has a higher sampling rate the sample delays can be determined with greater accuracy and there is no need for sample interpolation.

Just as for the broadside case, successive samples n are taken from a randomly selected channel but a delay is applied by placing them into the aggregate beamformer output sequence at index $m = n + d^{\theta}_{s(n)}$.

$$B^{\theta}_{\text{abf}}(n + d^{\theta}_{s(n)}) = X_{s(n)}(n). \qquad (5)$$

Because of the random time delays $d^{\theta}_{s(n)}$, this process may assign more than one sample n to the same output index m (causing a *collision*) and it may leave some output indices without any data assigned (causing *voids*). Statistical analysis shows that collisions/voids occur about 35% of the time (except for the broadside case). This increases the residual noise level of the aggregate beamformer only slightly.

Handling collisions and voids is an important aspect of implementing the aggregate beamformer but is beyond the scope of this presentation. One approach to handling colllisions is to randomly choose one of the colliding samples to keep. Another approach is to compute the average of colliding samples but this negates one of the advantages of the aggregate beamformer - it requires no arithmetic operations (except for the decimation filtering.) A simple and effective way to handle voids is to replicate the previous data value.

Figure 2 (top) compares the aggregate beamformer to the conventional DS beamformer. The residual noise power (lower frame) is essentially constant off-beam and is substantially reduced at the on-beam direction. (Optimization techniques beyond the scope of this presentation have been used for this example.)

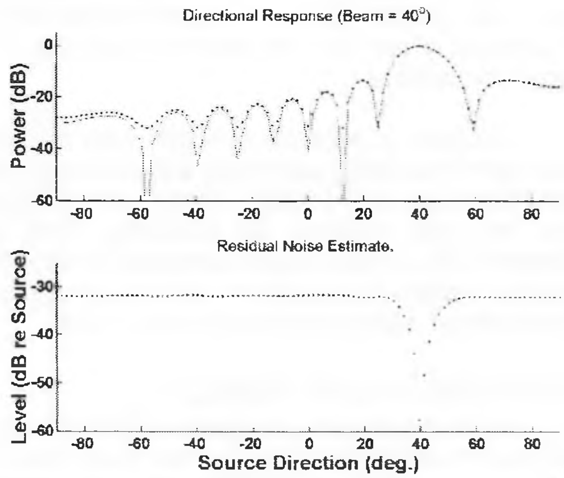

Fig. 2. Directional response (top) and residual noise (bottom) of aggregate beamformer (dot) steered to 40•: A 64-element square array is used with M=256 over sampling factor. The corresponding directional response for a conventional DS beamformer (top, solid) is shown for comparison.

REFERENCES

Havelock, D.I., "Aggregate beamformer for use in a directional receiving array," (Patent pending).