# FREE EDGE CONDITION ON POROUS MATERIALS

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## 1. INTRODUCTION

The free edge condition is encountered when the surface or contour of an elastic domain interacts freely with the surrounding acoustic domain. For an air-saturated open-cell porous layer, this condition poses some problems since the acoustic impedance of the surrounding domain is of the same order of magnitude than the one of the porous domain. In this case, the fluid loading of the surrounding domain on the porous layer cannot be neglected.

In the mixed Biot *u-p* poroelastic formulation [1], the free edge condition is classically modeled by imposing the acoustic pressure on the free surface of the porous sample to zero [2]. This implementation of the free edge condition does not seem to accurately model reality as the acoustic pressure will vary on the free surface. In this paper, a different implementation of the free edge condition on an air-saturated open-cell porous layer is proposed. It consists in applying an impedance radiation condition on the free surface of the sample. This new implementation of the free edge condition is also derived under the mixed Biot *u-p* poroelastic formulation.

The paper is structured as follow. First, the finite element model used in this research is detailed. Second, the different hypotheses and the finite element implementation of the free edge condition are discussed. Third, the experimental setup used to validate the numerical simulation is presented. Finally, the comparison between the numerical and experimental results are presented and discussed.

## 2. FINITE ELEMENT MODEL

As stated previously, the poroelastic finite element model used in this paper is based on the mixed Biot *u-p* formulation. To reduce computation time and memory usage and to enhance the convergence rate, both an axisymmetric formulation and high order Lagrange polynomial shape functions are used [3]. Therefore, similarly to the axisymmetric version of the displacement Biot *u-U* formulation [4], a transformation from cartesian to cylindrical coordinates is applied to the original *u-p* formulation, followed by an integration along the θ-axis. This yields the following weak integral equation, $\forall(\delta u_i, \delta p)$:

$$2\pi\int_{\Omega}\left[\hat{\sigma}_{ij}^{s}(u)\varepsilon_{ij}^{s}(\delta u)-\omega^{2}\tilde{\rho}_{s}u_{i}\delta u_{i}\right]rdrdz$$
$$+2\pi\int_{\Omega}\left[\frac{\phi^{2}}{\omega^{2}\tilde{\rho}_{22}}p_{,i}\delta p_{,i}-\frac{\phi^{2}}{\tilde{R}}p\delta p\right]rdrdz$$
$$-2\pi\int_{\Omega}\tilde{\gamma}\delta(p_{,i}u_{i})rdrd\,z\underbrace{-2\pi\int_{\Sigma}\hat{\sigma}_{ij}(u)n_{j}\delta u_{i}\,dS}_{I_{\Sigma}^{S}}, \quad (1)$$
$$\underbrace{+2\pi\int_{\Sigma}\left[\tilde{\gamma}u_{i}n_{i}-\frac{\phi^{2}}{\omega^{2}\tilde{\rho}_{22}}p_{,i}n_{i}\right]\delta p dS}_{I_{\Sigma}^{f}}=0$$

where $\Omega$ and $\Sigma$ are the volume and surface of the porous aggregate, respectively. $dS$ is an elementary surface on $\Sigma$, $u$ is the solid phase displacement field, and $p$ the fluid phase acoustic pressure field of the porous aggregate. $I_{\Sigma}^{S}$ and $I_{\Sigma}^{f}$ are respectively the surface integrals for the solid and fluid phases. The other parameters are detailed in references [1,5]. This formulation allows for the modeling of a 3D axisymmetric geometry with a 2D meshing, with $(r, z)$ and $(\xi, \eta)$ the global and local coordinates of the 2D meshing.

## 3. FREE EDGE IMPLEMENTATION

The following relations need to be applied on the free surface of the porous sample in order to model the free edge condition:

$$\begin{aligned} &p = P_{rad} \\ &P_{rad} = -j\omega z_{o}\left[(1-\phi)u_{n}+\phi U_{n}\right] \\ &\hat{\sigma}_{ij}^{s}(u)n_{j}^{P} = -(1-\phi)P_{rad}n_{i}^{P}+\phi\frac{\tilde{Q}}{\tilde{R}}pn_{i}^{P} \end{aligned} \quad . \quad (2)$$

The first relation indicates that the acoustic pressure on the surface is equal to the radiated pressure in an infinite acoustic medium. A good approximation of this radiated pressure is to consider normal radiation on the free surface, as it is done in the second relation. Finally, the last relation shows that the *in vacuo* stress tensor is the sum of the total stress on the solid phase, $-(1-\phi)P_{rad}n_i^P$, and the stress on the fluid phase, $\phi(\tilde{Q}/\tilde{R})pn_i^P$.

Substituting the first and third relation of eq. (2) in $I_\Sigma^S$ and considering that $\tilde{Q}/\tilde{R} = (1-\phi)/\phi$ yields:

$$I_\Sigma^S = 0 . \qquad (3)$$

Similarly, substituting the second relation in $I_\Sigma^f$ and considering that $\phi(1+\tilde{Q}/\tilde{R}) = 1$ gives:

$$I_\Sigma^f = \frac{2\pi R}{j\omega z_o}\int_\Sigma p\delta p dz . \qquad (4)$$

## 4. EXPERIMENTAL SETUP

To compare the measurements with the numerical results, disk shape samples need to be used so that the entire setup can be considered as being symmetric along its central axis. Figure 1 illustrates this setup. The porous sample is bonded in between two large rigid plates. There is a hole in the middle of the second plate from where a normally incident plane wave, generated by the standing wave tube, can excite the sample. Also, the circumference of the sample can interact freely with the surrounding acoustic domain. The sample as a diameter of 99 mm and a thickness of 76 mm. The material used for the measurements is a elastic foam whose properties are shown in Table 1.

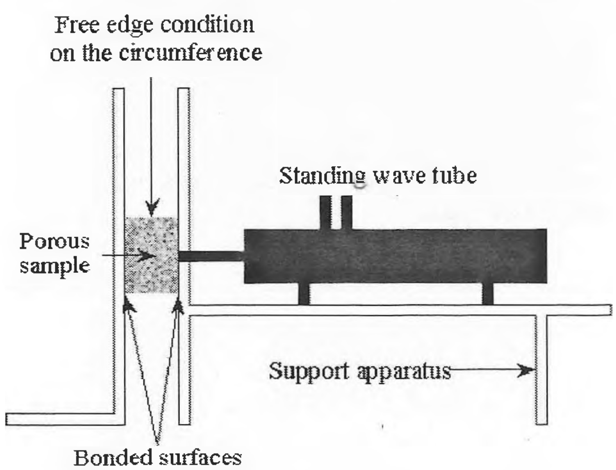


Figure 1. Experimental setup where a disk shape porous sample is excited by a normally incident plane wave. The circumference of the sample can interact freely with the surrounding acoustic domain.

## 5. RESULTS AND DISCUSSION

In Figure 2, the measurements and the numerical results for the elastic foam are compared. As it can be seen, there is a relatively good agreement between the two. This shows the accuracy and effectiveness of the modeling. Similar results have been obtained for other materials that are relatively isotropic. For certain materials, the agreement is not as good as shown previously, but this could be explained by the presence of a certain anisotropy.

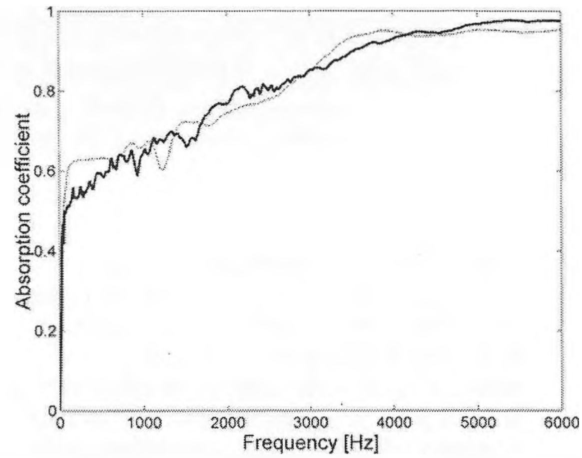


Figure 2. Absorption coefficient vs. frequency. —: measurements with the special setup, - - -: numerical results with the impedance condition on the circumference.

Table 1 - Material properties for the elastic foam

| Property | Unit | Value |
|---|---|---|
| E | N/m$^2$ | 93 348 |
| $\sigma$ | Ns/m$^4$ | 12 569 |
| $\rho_1$ | Kg/m$^3$ | 8.9 |
| $\nu$ | | 0.44 |
| $\phi$ | | 0.99 |
| $\alpha_\infty$ | | 1 |
| $\Lambda$ | μm | 56 |
| $\Lambda'$ | μm | 319 |
| $\eta_s$ | | 0.06 |

## REFERENCES

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## ACKNOWLEDGEMENTS

The authors wish to thank N.S.E.R.C. Canada and F.C.A.R. Quebec for their financial support.