SCATTERING OF A PLANE WAVE BY A CYLINDER WITH SURFACE IMPEDANCE THAT VARIES WITH POSITION

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ABSTRACT

In this work 'the source simulation technique' was used to calculate the scattering of a plane wave by a circular cylinder with surface impedance that varies with position. The basic idea of the source simulation technique is to replace the scatterer by a system of simple sources located within the envelope of the original body. The efficiency of the method was verified through the comparison between numerical results and experimental data. The calculation of the scattering was performed for the variants of the method: the single-layer method and the one-point multipole method. The matching between theoretical and experimental results was in the overall good, despite some occasional discrepancies.

SOMMAIRE

Dans cet ouvrage, la technique de simulation de source a été utilisée pour calculer la diffusion d'une onde plane par un cylindre avec une impédance de surface qui varie avec la position. L'idée de base de la technique de simulation de source est de remplacer le diffuseur par un système de sources simples localisées à l'intérieur de l'enveloppe du corps original. L'efficacité de cette méthode a été vérifiée en comparant les résultats numériques et des données expérimentales. Le calcul de la diffusion a été faite par les variantes de cette méthode: la méthode de couche unique et la méthode multipôle à un point. Les résultats théoriques et expérimentaux étaient généralement comparables, en dépit de divergences occasionelles.

1. INTRODUCTION

The mathematical treatment of radiation and acoustic scattering represents a very old and much studied problem of mathematical physics. Both phenomena were first treated more than a century ago by Lord Rayleigh [1,2]. Rayleigh suggested that the sound field radiated from a transverse vibrating rigid body is built up from spherical wave functions. This is the basic idea of the source simulation technique, that is, to replace the vibrating body by a system of radiating sources, which act in an equivalent way on the surrounding medium as the original body. The sources are located inside the radiator, and the problem consists of finding the source amplitudes. As long as the source amplitudes are known, the pressure and the velocity can be mapped at each point in the field.

This work was aimed at 1) showing the formulation of the scattering problem with the source simulation technique and 2) presenting its variants, the one-point multipole method and the single layer method. These variants were employed in the calculation of the scattering by a rigid cylinder, an absorbent cylinder, and by a cylinder with variable surface impedance. The cylinder was always considered infinite. The numerical results thus obtained were compared to experimental data collected in an anechoic chamber.

2. DESCRIPTION OF THE RADIATION AND SCATTERING PROBLEM

Consider the scatterer or radiator with surface **S**. The interior from **S** is called $\mathbf{S_i}$ and the exterior field $\mathbf{S_e}$. The normal surface **n** is directed to the exterior field $\mathbf{S_e}$. Throughout this article, only the exterior problems will be treated [3].

In the exterior field, the complex sound pressure p should satisfy the Helmholtz equation

$$\Delta p + k^2 p = 0 \tag{1}$$

where, $k = \omega/c$ is the wave number, ω is the circular frequency, c is the speed of sound and Δ is the Laplace operator. All the variables as functions of time should obey the function $e^{j\omega t}$. As long as the sound radiation in a free three-dimensional space is considered, the pressure p should also satisfy the Sommerfeld radiation condition [4]:

$$Lim_{r\to\infty} r\left[\frac{\partial p}{\partial r}+jkp\right]=0 \qquad (2)$$

which could be considered as the boundary condition at infinity. Here,

$$r=|\vec{x}| \quad , \quad \vec{x}=(x_1,x_2,x_3)$$

and r is a position vector and denotes the distance from the center to each point x in the field. Ochmann [5] termed the solutions of Eq. (1) satisfyimg the boundary condition of Eq. (2) as radiating wave functions. Typical functions that represent this class are called spherical wave functions ([5], [6]), which are generated when the solution of the wave equation is obtained in spherical coordinates. For the sake of simplicity, radiating wave functions will be called *sources* [5]. A complete description of the problem requires a description of boundary conditions on the surface of the radiator or scatterer. The Neumann boundary conditions will be considered here. In this case the normal velocity, v_n, and the gradient of the pressure

$$\partial p/\partial n=-j\omega\rho v_n \qquad (3)$$

on S are described. In Eq. (3), ρ is the density of the surrounding medium S and $\delta/\delta n$ is the derivative in the direction of normal **n** into the exterior field S_e. The problem of acoustic radiation is obtained if the normal velocity considered on the surface of the body is different from zero $v_n \neq 0$. Equation (3) represents an inhomogeneous boundary condition. Equations (1) and (2) describe the radiation problem for the radiated pressure p. With respect to the scattering problem, one should consider the incident wave p_i, which on its propagation encounters the surface S, then generating the scattered wave p_s. The scattering problem for the scattered

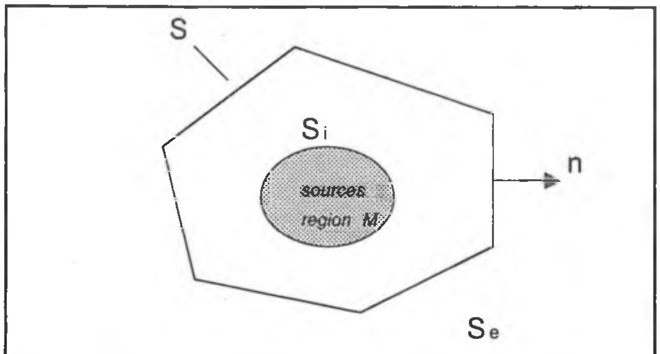

Figure 1. Geometry of the radiation and scattering problem (Ochmann, 1999).

wave p_s is described by Eqs. (1) and (2), but the pressure p should be accordingly substituted by pressure p_s in both equations. Considering again the Neumann boundary value problem, the outcome is that for a totally rigid body, the surface velocity should be equal to zero, that is, $v_n = 0$. That is,

$$\partial p/\partial n=0 \qquad (4)$$

In Equation (4) the pressure p represents the total pressure $p_t = p_i + p_s$. Equation (4) thus represents a homogeneous boundary condition. The scattering problem can then be formulated as a radiation problem. One should then consider velocity v_i of the incident wave p_i on the surface S. If surface S vibrates with negative normal velocity ($-v_i$), the radiated pressure is identical to the pressure p_s, originated from the incidence of p_i on S [4]. As a consequence, it is possible to write instead of Eq. (4)

$$\partial p/\partial n=-j\omega\rho(-v_i) \qquad (5)$$

for the scattering problem. Equation (5), similar to Eq. (3), represents an inhomogeneous boundary condition. Equations (1), (2) and (5) thus describe the scattering problem in an equivalent way to a radiation problem with respect to the scattered wave p_s.

3. Principle Of The Source Simulation Technique

The principle of the method is based on a treatment of the radiation problem or the scattering problem through a system of radiating sources, which should be chosen so that they reproduce as well as possible the sound field generated by the body of Figure 1. In the space previously occupied by the body S, the sources can now be found in region **M** shown in Figure 1. The sources are taken as point sources, and therefore do not represent an obstacle to the sound field. As a consequence the field generated by each one can be summed without taking into consideration interference effects. As the source amplitudes are known, the sound field can then be easily calculated through the sum of the fields generated by each source individually. The true problem consists then in finding the sources that can best replace the original body. As a consequence, two important questions arise:

1) Which is the type of source to be used and how should they be placed inside the body?
2) Which optimization method should be employed for the results?

Mathematically the problem is based on representing the sound field by summing up the contributions of the individual sources

$$p = \sum_{q=1}^{Nq} \sum_{m=-\infty}^{m=+\infty} A_{q,m} \phi_{q,m} \quad (6)$$

where p represents the scattered pressure or the radiated pressure in the field; $A_{q,m}$ is the complex source strength of the q^{th} source at a point x_q in the field; m is the order of each source and $\Phi_{q,m}$ is the sound field generated by the sources. In Eq. (6) $\Phi_{q,m}$ could also be called the source function. Equation (6) intrinsically has the condition that each field can be represented by a sum of functions of the type $\Phi_{q,m}$. This is naturally the case, only if all functions $\Phi_{q,m}$ satisfy the wave equation and if they form a complete function system. This condition is certainly satisfied if $\Phi_{q,m}$ represents, for example, the field generated by a monopole, dipole or quadrupole. As no difficulty has been noted by other authors ([5], [7], [8], [9], [10]) when multipole sources were used for the reconstruction of the acoustic field, the same procedure will be used in the present work. In other words, the multipole sources will be used to represent the radiation or scattering problem of the original body.

Two distinct situations pose themselves:

1) one can use a variable order multipole localized in a single point inside the body, that is, in Eq. (6) $Nq = 1$ and M is very large, or
2) one can use only monopole sources positioned in several points inside the body, which renders Nq very large and $M = 1$ in Eq. (6).

One can also have a combination of both extreme cases presented in a) and b), that is, to use a multipole positioned in several points. Together with the choice of the type and the positioning of the sources, the choice of the optimization criterion also imposes a fundamental question for the use of the source simulation technique. The error derived from this optimization should be minimized. Several methods can be used to that end, such as the null field method, the collocation method, or Cremer's method. The least square minimization method has been used in this work.

4. Source Function System

If a source alone generated the sound pressure $A_{q,m}$ $\Phi_{q,m}$, then the sum

$$p = \sum_{q=1}^{Nq} \sum_{m=-M}^{m=+M} A_{q,m} \phi_{q,m} \quad (7)$$

should approximate the original field as well as possible. Each of the individual source functions $\Phi_{q,m}$ is supposed to meet the radiation condition in Eq. (2) and the wave equation in Eq. (1) in the exterior field domain $\mathbf{S_e}$. When these conditions are satisfied, one can take them from any complete function system. In practical terms, source functions, which can be written in conventional coordinate systems, can be used: spherical coordinates (for three-dimensional problems) and cylindrical coordinates (for two-dimensional problems).

The velocity generated by the source function system at the radiator surface is calculated by inserting Eq. (7) in Eq. (3). For reasons of simplicity, we have taken the one-point multipole method, so that $q = 1$

$$v_n = -\frac{1}{j\omega p} \cdot \sum_{m=-M}^{m=+M} A_m \frac{\partial \phi_m}{\partial n} \quad (8)$$

Eq. (8) can be rewritten, since $k = \omega/c$, as

$$v_n = -\frac{1}{\rho_0 c_0} \cdot \sum_{m=-M}^{m=+M} A_m Z_m \quad (9)$$

where,

$$Z_m = (1/jk)\partial\Phi_m / \partial n$$

is a function defined in a similar way as in Heckl [8]. For example, the function Z_m in the commonly cylindrical coordinates for two dimensions is given by

$$Z_m = +\frac{1}{j}\left[H_m^{'(2)}(kR)e^{+jm\phi}\frac{\partial r}{\partial n} + \frac{m}{k} H_m^{(2)}(kR)e^{+jm\phi}\frac{\partial \phi}{\partial n}\right] \quad (10)$$

where,

R is the radius from the radiated body;

$H_m^{'(2)}(kR)$ and $H_m^{(2)}(kR)$

are the derivative of the Hankel function of the second order; and the Hankel function of the second order, respectively. For the scattering problem, the calculations go in a similar way. The total velocity generated on the scatterer surface is given by

$$v_{t(n)} = v_1 + v_2 \quad (11)$$

where $v_{t(n)}$ is the total generated velocity on the scatterer surface in the normal direction, v_1 is the velocity from a normally incident wave at the surface of the body, and v_2 is the scattered velocity when the body is present in the field

$$v_{t(n)} = -\frac{1}{j\omega p}\frac{\partial(p_t)}{\partial n} = -\frac{1}{j\omega p}\left[\frac{\partial(p_i)}{\partial n} + \sum_{q=1}^{N_q}\sum_{m=-M}^{m=+M} A_{q,m}\frac{\partial(\phi_{q,m})}{\partial n}\right] \quad (12)$$

where p_t is the total pressure on the scatterer surface and is given by

$$p_t = p_i + p_s \quad (13)$$

$$p_t = p_i + \sum_{q=1}^{Nq}\sum_{m=-M}^{m=+M} A_{q,m}\phi_{q,m} \quad (14)$$

where p_i is the pressure from the incident wave and p_s is the scattered pressure in the field. For reasons of simplicity, we have taken again the one-point multipole method, so that $q = 1$,

$$p_t = p_i + \sum_{m=-M}^{m=+M} A_m \phi_m \tag{15a}$$

and

$$v_{t(n)} = -\frac{1}{j\omega p}\frac{\partial(p_t)}{\partial n} = -\frac{1}{j\omega p}\left[\frac{\partial(p_i)}{\partial n} + \sum_{m=-M}^{m=+M} A_m Z_m\right] \tag{15b}$$

The function Z_m is the same as in Eq. (10) for the two-dimensional problem with cylindrical coordinates. If the incident wave is a plane wave in cylindrical coordinates

$$p_i = p_0 e^{-jkR\cos(\phi)} \tag{16}$$

so, $v_{t(n)}$ is rewritten as

$$v_{t(n)} = \frac{p_0}{\rho_0 c_0}\cos(\phi)e^{-jkR\cos(\phi)} - \frac{}{\rho_0 c_0}\sum_{m=-M}^{m=+M} A_m Z_m \tag{17}$$

The requirement that the velocity distribution given by Eqs. (9) and (17) generated by the sources at the surface should approximate the prescribed normal velocity as well as possible leads to a linear system of equations through which the complex source strength will be determined.

5. Optimization Criteria

Several methods can be used in order to minimize the error in the surface velocity approximation [10]. The least squares minimization method has been used in this work. The technique consists in minimizing the surface integral error

$$\int \left|v_{t(n)} - v_b\right|^2 dS = Min \tag{18}$$

which sums the errors generated in the approximation of the surface velocity. In Eq. (18) **S** is the surface of the scatterer, **dS** a surface element and $v_{t(n)}$ is the velocity generated by the source simulation technique. For the special case of scattering from a rigid body, the surface velocity is zero, so that $v_b = 0$.

$$\int_S \left|v_{t(n)}\right|^2 dS = Min \tag{19}$$

The velocity has the same form as in Eq. (15b) for the one-point multipole method. The system of equations for the determination of the sources strength A_m is obtained through the calculation of the partial derivative of the integral in Eq. (19) with respect to A_m, and letting the result equal to zero

$$\frac{\partial}{\partial A_m}\left(\int_S \left|v_{t(n)}\right|^2 dS\right) = 0 \tag{20}$$

The solution of the linear system of Eq. (20) gives us the sources strength A_m which when substituted in Eqs. (15a) and (15b) allow the calculation of the sound pressure and the sound velocity for each point in the acoustical field. Thus, the problem is perfectly solved. For the somewhat general case, that the body is not rigid but has a constant relation on the whole surface between the total sound pressure and the total sound velocity in the direction of the normal, this leads Eq. (18) to

$$\int_S \left|v_{t(n)} - \frac{p_t}{Z}\right|^2 dS = Min \tag{21}$$

where Z is the surface impedance of the scatterer.

The condition imposed on impedance is that it should not have lateral couplings, that is, it should be locally reacting. This means that one part of the surface is not aware of the motion of another part, and the reaction of one part of the surface is proportional to the local pressure at that point. This condition indicates the non-inclusion of elastic surfaces (for example, surfaces where flexion waves are possible). This extremely rigid limitation should be verified in each case. Elastic bodies, as for example a thin-walled cylinder immersed in water, certainly do not satisfy it. For porous materials (for example foam) one can in principle assume that for air borne sound there is no lateral coupling, that is, the materials are locally reacting. In the same way, we can calculate the radiation problem with the least squares technique. This leads to the surface integral

$$\int_S \left|v_b - v_n\right|^2 dS = Min \tag{22}$$

and again the surface error should be minimized. In Eq. (22) v_b is the velocity of the vibrating body and v_n is the velocity generated from the sources. For the one-point multipole method and for the two-dimensional case in cylindrical coordinates v_n is the same as in Eqs. (9) and (10). As in the case of scattering, if the partial derivative in Eq.(22) is calculated with respect to source strength A_m and making the result equal to zero, one has a system of linear equations through which the complex sources strengths are determined. Substituting them in Eqs. (7) and (9) we have the pressure and the velocity at each point in the acoustic field.

6. Calculation Of Scattering Field by Source Simulation Technique

The next issue to be addressed is the problem of calculating sound scattering for an infinite circular cylinder, in which the random distribution of the surface impedance is considered. The calculation will be performed for the one-point multipole method and for the single layer method.

6.1 One-Point Multipole Method

In this case the approach includes the choice of multipole expansions up to high orders at only one location. Using the symmetry of the circular cylinder, the location point coincides with the center of the cylinder. The condition of an infinite cylinder means that the problem is treated independently of the axial direction, that is, $\delta/\delta z = 0$. It must also be pointed out that only plane harmonic waves are considered in this work.

The total pressure p_t can be written as a sum of the incident plane wave p_i and the scatterer wave p_s

$$p_t = p_i + p_s \tag{23}$$

The incident plane wave traveling in a direction perpendicular to the cylinder's axis is given by $p_i = p_0 e^{-jkx}$, and p_0 is the amplitude. The scatterer wave p_s is given by Eq.(7) and Eq. (15), so that

$$p_t = p_0 e^{-jkr\cos(\phi)} + \sum_{m=-M}^{m=+M} A_m H_m^{(2)}(kr) e^{jm\phi} \tag{24}$$

In cylindrical coordinates, $x = r\cos(\Phi)$, and r is the distance between the center of the cylinder and any point in the surrounding medium.

The expression for the velocity is obtained through Eq. (3), since the normal direction coincides with the radial direction and on the surface $r = R$.

$$v_{t(n)} = \frac{p_0}{Z_0}\cos(\phi) e^{-jkR\cos(\phi)} \frac{1}{Z_0} \sum_{m=-M}^{m=+M} A_m H'^{2}_{m}(kR) e^{+jm\phi} \tag{25}$$

where $Z_0 = \rho_0 c_0$ is the specific acoustical impedance of the air and

$H_m^{'(2)}(kR)$

is the second order derivative of Hankel function. All time varying quantities should obey the time dependence $e^{+j\omega t}$. As the exponential factor is shared by all field quantities, it can be omitted.

As mentioned earlier, the surface impedance is assumed to be locally reacting. Therefore, the boundary conditions for each surface element and for each angle Φ on the surface is,

$$v_{t(n)} = p_t / Z \tag{26}$$

where Z is the surface impedance.

In this work it is considered that the impedance is randomly distributed on the surface. Hence, the impedance could be infinite, that is, for a rigid surface in the interval $\Phi_0 \le \Phi \le \Phi_1$, or could be finite, assume the value Z in the interval $\Phi_1 \le \Phi \le \Phi_2$. The impedance Z was measured with the standing wave apparatus for a 5 cm-thick foam, and will be used in the numerical calculation as inputs for the solution of the problem.

Considering the optimization criterion given by Eq. (21), we have

$$R\left[\int_{\phi_0}^{\phi_1} \left|v_{t(n)}\right|^2 d\phi + \int_{\phi_1}^{\phi_2} \left|v_{t(n)} - \frac{p_1}{Z}\right|^2 d\phi\right] = Min \tag{27}$$

Differentiating Eq. (27) with respect to the unknown source strength and making the result equal to zero, we obtain a system of linear equations with them the complex sources strength, that is, the solution of the posed problem, can be find.

6.2 Single Layer Method

In this method, several monopole sources are positioned on an auxiliary surface. The auxiliary surface is placed inside the body. Note that the auxiliary surface should not coincide with the surface of the original body. If the auxiliary surface coincides with the surface of the body, the problem cannot be solved by the source simulation technique. Instead other methods such as boundary element method (BEM) need to be applied. The auxiliary surface has the same form as the surface of the body being studied, that is, the circular cylinder. For the total pressure we have

$$p_1 = p_0 e^{-jkR\cos(\phi)} + \sum_{q=1}^{N_q} A_q H_{0,q}^{(2)}(kr) \tag{28}$$

where r is the distance between a point with polar coordinates (R, Φ) on the cylinder surface and a source point q with the polar coordinates $(r_{(q)}, \Phi_{(q)})$. R is the radius of the circular cylinder. The cosine law gives us

$$r = \sqrt{R^2 + r_{(q)}^2 - 2Rr_{(q)}\cos(\phi_{(q)} - \phi)}$$

and the normal component of the velocity on the surface at a

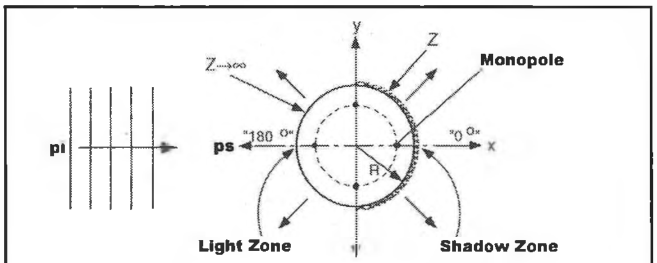

Figure 2. Single Layer Method for the rigid and absorbing circular cylinder.

point (R, Φ) is

$$v_{t(n)} = \frac{p_0}{Z_0}\cos(\phi)e^{-jkR\cos(\phi)} - \frac{1}{Z_0}\sum_{q=1}^{N_q} A_q H'^{(2)}_{0,q}(kr)r'(R) \quad (29)$$

where, $r'(R) = \partial(r)/\partial(R)$ and $H_0^{'(2)}(kR)$ is the second order derivatives of the Hankel function. Inserting Eqs. (28) and (29) into Eq. (27), the partial derivatives with respect to the unknown source strength are equated to zero, and then we obtain a system of linear equations similar to Eqs. (36). This system of equations give us the complex sources strength, that is, the solution of the problem.

7. Experimental Methodology

The experiments were performed in an anechoic chamber with a 3 m long rigid cylinder with a radius of 15 cm (see Figure 3). The cylinder surface was covered with a 5-cm thick porous absorbing material. The impedances of the absorbing material were measured for different frequencies (200-8000 Hz) by a standing wave apparatus. The absorbing cylinder was covered for half of its perimeter with a metal plate, thus resulting in a half-rigid/half-absorbing cylinder. This characterization depends on the face of the cylinder exposed to the incident wave. The sound was generated by a noise generator in 1/3 octave bands and after being amplified it was irradiated through a loudspeaker. The sound was measured by a microphone mounted on a turning table which could face either the shadow zone or the light zone. The sound pressure levels were measured at each 10° of approach of the turning table, first without the cylinder in the field and then with the cylinder in the field. The difference between these measurements gives us the sound attenuation due to the presence of the cylinder, which is dependent on the frequency, on the surface impedance, and on the distance from the microphone (measuring point) to the center of the cylinder.

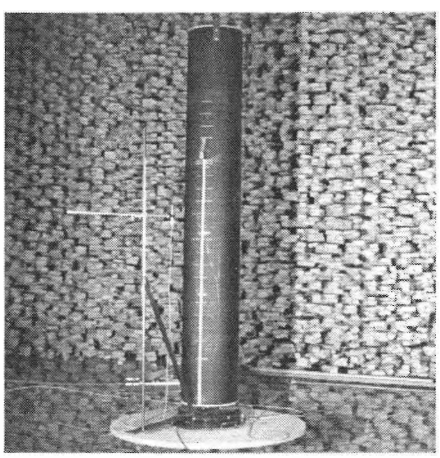

Figure 3. The rigid cylinder and the turning table.

8. Results And Discussion

The verification of the efficacy of the source simulation technique for the calculation of acoustical fields was by comparing the numerical results with the experimental data. In this sense, sound attenuation (shadow zone) produced by the presence of the cylinder in the field was both measured and calculated. It is impossible to present all the results obtained due to the great number of factors involved, such as frequency, distance from the point of measurement to the center of the cylinder, position angle, and surface impedance. Only some of the results obtained for the 1) rigid cylinder, 2) absorbing cylinder, and 3) absorbing and rigid, will be presented here. The results were calculated using the single-layer method, which were not essentially different from the ones obtained using the one-point multipole method.

Figures 4 to 6 show that although there is a very good agreement between the numerical results obtained with the source simulation technique and the experimental data, some discrepancies can be noted. Possible causes are discussed below. One possible reason for the differences in the values obtained for the sound attenuation in Figures 6 to 8 is that the calculation was undertaken for a bi-dimensional problem, while the measurements were performed in a three-dimensional model. The numerical calculation is always valid for a single frequency. However, the experiments used the sound generated by a band of 1/3 octaves. This fact can lead to inaccuracies in the numerical calculation, as the acoustical field generated by the scatterer changes too rapidly with the frequency and the angle of the measurement point. This numerical difficulty can be avoided if one takes the mean of the results for several frequencies inside each band of frequencies. In other words, the central frequency of the band of interest is considered and the frequencies below and above the central frequency are harmonically calculated. Several numerical tests were performed and one can conclude that a good approximation of the theoretical and experimental results can be obtained when the mean value was calculated out of 5 frequencies. However, one should not discard the possibility that, in some cases, the mean value should be calculated from a larger number of frequencies, especially in the high frequency range.

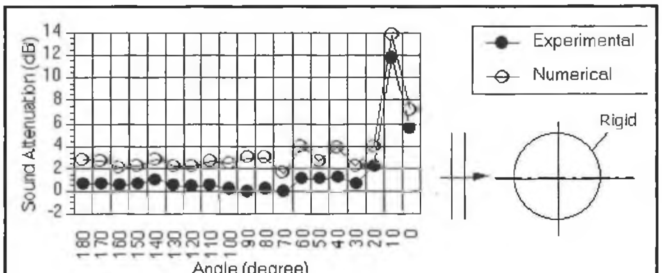

Figure 4. Scattering from a rigid cylinder: kR=8.66; distance from a point in the field to the center of the cylinder: 58 cm, number of monopoles: N=35.

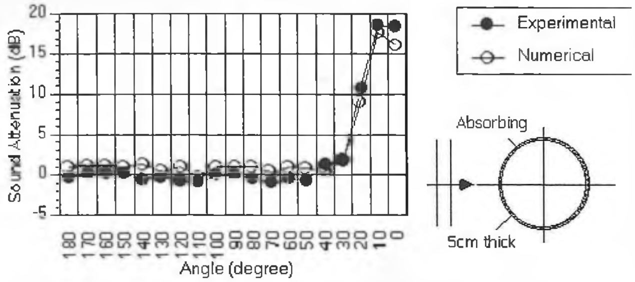

Figure 5. Scattering from an absorbing cylinder: kR=23; distance from a point in the field to the center of the cylinder: 58 cm, number of monopoles: N=138, surface impedance: Z=1065.82 – j 29.81.

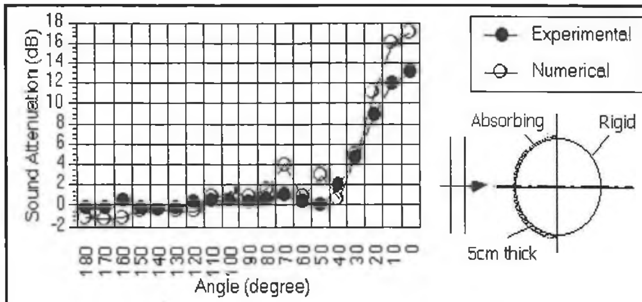

Figure 6. Scattering from a half-absorbing and half-rigid cylinder: kR=14.65; distance from a point in the field to the center of the cylinder: 47 cm, number of monopoles: N=59, surface impedance: Z=250.3 – j 200.

The influence of possible reflections from chamber walls was tested if the sound attenuation in the shadow zone, ($\Phi = 0$) was above 20 dB, by placing the cylinder in other positions as well as at longer distances from the walls. However, the new measurements did not show any significant difference from the results obtained previously. Equation (27) is based on the hypothesis that the surface impedance is locally reacting. This hypothesis was also considered for the 5-cm thick foam with which the cylinder was covered, thus generating an absorbing cylinder. In order to confirm that the foam was locally reacting, fissures were made in the surface of the foam. But the sound pressure levels measured afterwards showed no significant modification when compared to the values obtained previously. One may thus conclude that the material used behaves as locally reacting. An ideal agreement between calculation and measurement would be obtained if the index M in Eq. (24) or the index N_q in Eq. (28) grow without any bounds. That is, in practical terms, impossible because of the immense computing time. Numerical simulations have shown that a good agreement between calculation and measurement is found for $M_{max} = 4\pi R\lambda$, and for $N_{q(max)} = 6\pi R\lambda$, where R is the cylinder radius and λ is the wavelength. Exceptions to this rule are in some regions of the shadow zone, between $-10° \leq \Phi \leq +10°$.

Another important reason for the differences between the numerical results and the experimental data resides in the fundamental principle of the method, that is, not exactly reconstructing each surface element at the given boundary conditions, but to minimize the error through an integration, like in Eq. (18), over the whole perimeter of the body. Equation (18) corresponds to the optimization of the error in the surface velocity approximation in the least mean square procedure. Equation (18), and thus the source simulation technique allows the control of the error as they satisfy the boundary conditions for every computation. This is a very important characteristic of this method, mainly when an analytical solution to the problem is not available. For practical cases, however, it would be important to assure a controlled accuracy not only of the surface velocity as in Eq. (18), but also in the determination of the sound power. The use of an infinite number of sources would certainly allow the precise reconstruction not only of the surface velocity, but also of the sound power. It must be pointed out therefore, the reconstruction of the surface velocity is reasonably accurate due to the use of a finite number of sources (a maximum value for M and N_q). However, the finite source number is nt sufficient for the determination of the sound power. With this limitation, one has in hand a very efficient method for the reconstruction of the acoustic field.

9. Conclusions

This work has presented the study of the scattering for an infinite cylinder with variable surface impedance, both numerically and experimentally. The theoretical analysis used the Method of Source Simulation. This method has been frequently used in the last decade for the solution of purely theoretical and/or numerical radiation and scattering problems. Very few works are found in the literature which allow a comparison between the numerical results obtained with the source simulation technique and experimental data. The contribution of the present work was then to present a comparison between numerical and experimental results so as to evaluate the practical usefulness of the source simulation technique. An important practical property of the source simulation technique is its controlled accuracy: the error is directly determined as a discrepancy in the boundary conditions on the surface of the body in each specific case. This property is very important especially if analytical solutions are not available. The principle of the method is very simple. Further research still needs to be done to investigate the influence of the type of source, type of surface over which the sources are positioned (single-layer method), the possible existence of resonance frequencies, and the applicability of the method for more complex surfaces. The main drawback of the source simulation technique is the fact that rules for the positioning of the source surface are not known a *priori*. The positioning of the source surface and in consequence of the sources themselves is based on the experience of the programmer.

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