# On the Simplifications Used in Mobility Models to Predict Structureborne Power Flow in Wood Stud Walls with Direct-Attached Gypsum Board

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## 1. INTRODUCTION

The mobility approach is a comparatively simple method to compute structure borne power flow between plates and beams coupled at one, or more, well defined points. Implicit to these expressions are a number of assumptions. Simplifications are also made when modelling wood stud wall systems. Using ordinary point force mobilities Craik and Smith [1] predicted with good accuracy the structural power flow across a wood stud wall with direct-attached gypsum board on both sides when the screws on either side of the stud were aligned. This type of alignment cannot be assumed if the wall has resilient channels. Thus it is necessary to evaluate the mobility assumptions to determine the most appropriate method to model walls with resilient channels. In this paper, the first of two, assumptions regarding the vibration response of the stud and gypsum board in isolation are evaluated. The second paper [2] examines the power flow from the stud to the gypsum board as a function of number and location of the screws. The paper begins with a brief review of the mobility expressions.

## 2. MOBILITY MODEL AND EXPRESSIONS

Power flow from a beam (stud) to a point-connected plate (gypsum board) can be written as,

$$W_{12} = N \frac{v_o^2 \Re(Y_2)}{|Y_1 + Y_2|^2} \qquad (1)$$

where N is the number of fastening points, and $v_o$ is the velocity of the source. $Y_1$ is the mobility (inverse of impedance) for the source (stud), $Y_2$ is the mobility for the receiver (gypsum board), given by,

$$Y_1 = (2\rho b h c_B (1+i))^{-1} \qquad (2)$$

and,

$$Y_2 = \left(8\sqrt{(B\rho h)}\right)^{-1} \qquad (3)$$

where ρ is the bulk modulus, h is the thickness, b is the width, B is the bending stiffness, and $c_B$ is the bending wave speed. Equations 2 and 3 are for point forces located far from an edge of a semi-infinite system. In the limit that the excitation point is at an edge these equations must be multiplied by 4 and 8/3.5, respectively. The equations are strictly valid only for systems that behave as if they are infinitely thin – ones for which there is no local deformation at the drive point and there is no deformation of the volume due to the applied force

## 3. ASSUMPTIONS MADE DURING APPLICATION

All fasteners are center-located: It has been suggested [1] that for practical purposes the mobility of the gypsum board at all screw locations can be approximated by the mobility of a point near the center of a large plate. To test this assumption the mobility of a sheet of 16 mm type X gypsum board was measured at 9.5, 19, and 50 mm from an edge as well as at the sheet center.

Figure 1 indicates that each mobility curve approaches the theoretical value (equation 3) for a center location asymptotically but at a different frequency. A point farther from the edge satisfies the assumption at a lower frequency than a point that is closer. Screws into the stud at the top and bottom of the sheet are typically 50 mm from the edge and can be considered to be center located for frequencies above 800 Hz. Between 315 and 800 Hz the edge-location assumption works best. There are few modes in the gypsum board below 315 Hz and the mobility estimates become unreliable and are overestimated. Screws at a butt joint, which are typically 9.5 mm from the edge, should be considered edge-located throughout the building acoustics frequency range.

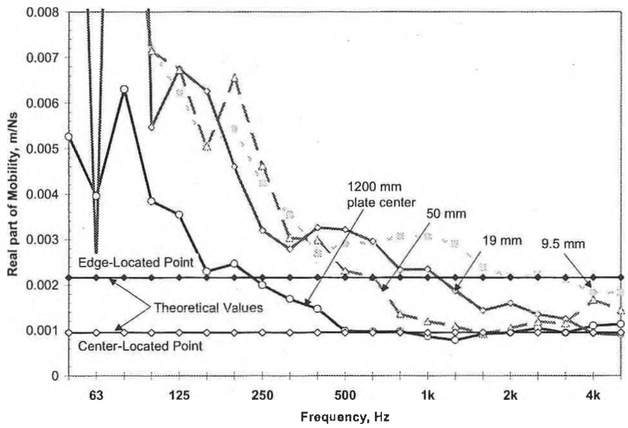


*Figure 1: Measured gypsum board mobility as a function of the distance from the edge of the sheet.*

In the mid and high frequencies the gypsum board mobility is not overly sensitive to location when the point is at least 50 mm from an edge. A similar trend was observed for the stud. Thus, the power flow from the stud to the gypsum board should be reasonably independent of location if the stud velocity is uniform.

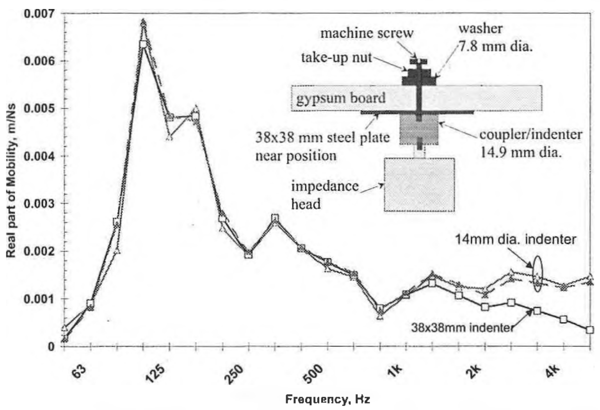


*Figure 2: Measured gypsum board mobility for two indenter sizes.*

Mobility is independent of contact area: Thin plate/beam theory assumes that the area of the drive point is infinitely small. Consequently, mobility expressions based on thin plate/beam theory may not be applicable if measured mobilities show a strong dependence on the size of the drive point, i.e. size of the indenter. To examine the sensitivity the drive point mobility of 16 mm gypsum board was measured using two indenter sizes – a 38 mm×38 mm ×2 mm plate (38 mm corresponds to the stud width) and a 14.9 mm dia. brass cylinder.

Figure 2 shows that the drive point mobility of gypsum board is a function of the contact area above about 1250 Hz where the real part of the mobility with a larger contact area (steel plate indenter) is about 3 dB lower than with the smaller 14.9 mm dia. indenter. Equation 1 indicates that less power will be transmitted from the stud to gypsum board for the larger drive point. This is consistent with that predicted from the advanced mobility theory of Petersson [3]. For a real wall this effect will reverse as the size of the indenter becomes significant compared to the spacing between excitation points.

<u>Plates and beams do not deform volumetrically:</u> This assumption states that the velocity - in magnitude and phase - is the same on both sides of the element. It is implicit in all expressions derived from thin plate/beam theory, e.g., equations 2 and 3.

Figure 3 shows a significant VLD across the depth of a stud for frequencies above about 2000 Hz that increases with frequency. The VLD peak at 630 Hz and 800 Hz between positions A1 and B1near the drive point might be caused by the local deformation and/or near field of the source. Figure 3 shows the assumptions of thin beam theory are not satisfied indicating that the theoretical value given by equation 1 will be a poor estimate above 2000 Hz. This is shown in Figure 4. For gypsum board the VLD was effectively zero and the mobility predicted by equation 2, which is shown in Figure 1, provides an accurate estimate in the mid and high frequencies.

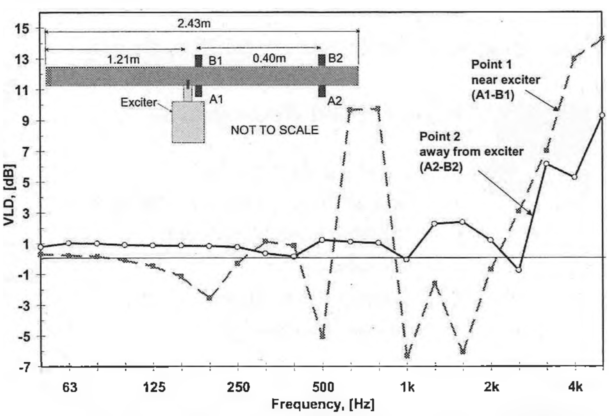


*Figure 3: Measured VLD across the depth of a 38x85 mm red cedar stud*

<u>Mobility is not appreciably affected by the boundary conditions of the plate and beam:</u> Because ordinary mobilities assume that the plate or beam is infinite, the effect of boundary condition is assumed negligible. To assess the effect, the average center mobility of the studs is compared for two boundary conditions. First, the stud is resiliently supported, which approximates the free-free conditions upon which the theory (equation 2) is based. Second, the stud is installed in the test frame but without attached gypsum board. This boundary condition is not free-free and is probably between clamped and simply supported. We restrict the mobility comparison to frequencies above 160 Hz where the point connection assumption is valid [2].

Above 630 Hz, the real part of the mobility for the two boundary conditions is quite similar suggesting that for mid and high frequencies differences in boundary condition are not important. The very close agreement above 2000 Hz may be due to the volumetric deformation where the mobility is largely determined by the behaviour near the drive point. Below 630 there is a noticeable difference – the stud mobility tends to be greater when installed in the wall and agrees better with theory (equation 2). This might seem counter intuitive but it should be recognised that installing the stud in the frame significantly increases the damping which diminishes the importance of individual modes which is important in the low frequencies where the there are few, and perhaps no, modes in some bands.

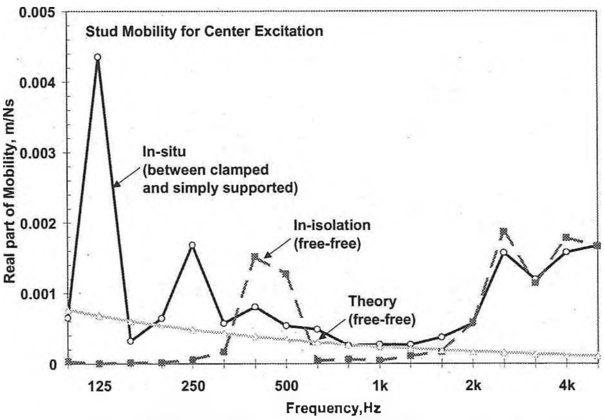


*Figure 4: Effect of boundary conditions on the mobility of 35x85mm red cedar studs.*

## 4. DISCUSSION AND CONCLUSIONS

The review of the primary assumptions used in ordinary mobility models of structure borne power flow indicate there will be a limited frequency range where the models based on thin plate theory can be applied. For the western red cedar studs considered here the upper frequency limit is determined by volumetric deformation of the stud. Volumetric deformation may not be important for other wood species; especially ones that have a comparatively high shear modulus or have knots. Other species and quality grades will be examined in subsequent phases of this project. Advanced mobility theories [3,4] account for the effect of volumetric and local deformation.

For fastening points located at least 50 mm from an edge the mid and high frequency mobility can be reasonably approximated by a center-mobility and that boundary conditions should have little effect. A low mode count prevented examination in the low frequencies.

The high frequency mobility of gypsum board is a function of the area of the drive point. Consequently, the gypsum board mobility measured with a small indenter may agree well with theory (Figure 1) but might be significantly different from the in-situ mobility seen by the stud where there the area over which the force is applied may be considerably larger (Figure 2). The effect of contact area on the mobility wood studs and gypsum board should be investigated further.

## 5. REFERENCES


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