A TWO-STAGE METHOD FOR DETERMINING THE POSITION AND CORRESPONDING PRECISION OF MARINE MAMMAL SOUNDS

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ABSTRACT

Today there is a concern that man-made sounds, such as that from sonar experiments, seismic operations and oil rigs, affect marine mammals. Detection and localisation of marine mammals will definitely support measures to reduce the possible detrimental effects. This paper presents a two-stage localisation method, which is applied to data collected with an array of five hydrophones moored to the seabed in the Bay of Fundy, Canada. The array forms a 14 by 14 km square with one hydrophone in the centre. The method makes use of the relative travel times of the mammal's sound to the four hydrophones at the square vertices with respect to the travel time to the central hydrophone. First, a good initial position is obtained using hyperbolic fixing. In the second step the solution is improved in an iterative process, where each iteration determines the least-squares solution of the set of four linearized equations for the measured relative travel times. Calculating the error ellipse from the covariance matrix of the solution provides the localisation accuracy. There are several parameters that affect the source position accuracy. These include the uncertainties in arrival times, sound speed and receiver positions. Their effect on the localisation accuracy is assessed. [Work supported by Royal Netherlands Navy]

RÉSUMÉ

Aujourd'hui, une question préocupante est de savoir si des sons d'origine artificielle tels que ceux générés par les systèmes sonars, les opérations sismiques ou les installations pétrolières peuvent affecter les mamifères marins. La détection et la localisation des mamifères marins est un atou indéniable afin de réduire d'éventuels effets indésirables. Cet article présente une méthode de localisation en deux étapes appliquée à des données collectées par une antenne de cinq hydrophones amarrés au fond marin dans la baie de Fundy au Canada. L' antenne forme un carré de 14 par 14 kilomètres avec un hydrophone placé au centre. La méthode utilise la différence relative de temps de parcours des cris des mamifères marins entre les hydrophones situés aux sommets du carré et le temps de parcours jusqu'à l'hydrophone situé au centre. Premièrement, une bonne estimation initiale de la position est obtenue grâce à une correction hyperbolique. Dans la deuxième étape, cette solution est améliorée grâce à un processus itératif où chaque itération donne la solution au sens des moindres carrés d'un ensemble de quatre équations linéarisées obtenues grâce aux temps de parcours relatifs mesurés. Le calcul de l'ellipse d'erreur à partir de la matrice de covariance de la solution donne la précision de la localisation. Plusieurs paramètres affectent la précision de la position de la source. Ceux-ci incluent l'incertitude sur les temps d'arrivé, la vitesse du son et la position des récepteurs. Leurs effets sur la localisation sont évalués. [Travail subventionné par la Marine Royale Hollandaise]

1. INTRODUCTION

Marine mammals rely on their vocalizations for orientation, communication and hunting. There is an increasing concern that man-made acoustic signals are harmful to these animals. This has resulted in an increased research effort on passive acoustic techniques for detection and localisation of the mammals. This article focuses on accurate localisation.

To promote research on this topic, a workshop on detection and localisation of marine mammals using passive acoustics was held at Dartmouth, Nova Scotia, Canada, 19-21 November 2003. A dataset of marine mammal vocalizations was provided.

This data set has been used to test the localisation procedure presented here. This procedure relies on basic concepts of geodesy:

- application of the elementary adjustment principles of least squares to combine redundant measurements in an optimal way (with unbiased minimum variance)

- confidence intervals to quantify the uncertainty of calculated positions.

The procedure requires a proper starting solution, which is obtained through hyperbolic fixing. Both concepts are applied assuming free propagation in an unbounded homogeneous medium, i.e., spherical propagation where the sound is assumed to propagate along straight paths. The method is applied to data collected on a symmetrical array of five omni-directional hydrophones moored to the seabed in the Bay of Fundy, Canada. The array forms a 14 by 14 km square with one hydrophone in the center. The localisation method uses all four available travel time differences, estimated with respect to the central hydrophone L.

Section 2 describes the approach taken for determining the relative travel times and their uncertainties. Section 3 presents the localisation procedure. Section 4 presents the results. Also, in Section 4, the effect of the uncertainty in the arrival times, the uncertainty in the sound speed and the uncertainty in the receiver positions on the localisation accuracy are assessed. It is followed by the conclusions in Section 5.

2. TRAVEL TIME ESTIMATION

Since the moment at which the sound was emitted is unknown, only relative arrival times are available for the localisation. The data provided for the workshop consist of three distinct types of sound: "gunshots", mid-frequency calls and low-frequency calls. The approach selected for determining the relative travel times depends on the characteristics of the signal. For this paper we will consider both the gunshots and the mid-frequency calls.

The gunshots are broadband high-amplitude transients of short duration, making it possible to estimate by eye the arrival time from the spectrogram. Also the uncertainty of the arrival time is estimated from the spectrogram. For the mid-frequency calls accurate arrival time estimation from the spectrogram is not possible due to the nature of the signal. Here a clip of the signal from the spectrogram of one hydrophone is used as a template to be matched with the spectrograms of the other hydrophones. Successively using the received signals for each of the hydrophones as the template, a set of travel time differences is obtained. Taking a weighted average provides the estimated travel time differences. Although this matched filtering gives considerable pulse compression, the estimated uncertainty in the arrival times of the mid-frequency calls is much larger than that for the gunshots. Table I and Table II present the estimated arrival times and their corresponding uncertainties.

Table I: Estimated arrival times and standard errors (σ_t) [s] for the five hydrophones (L,C,E,H,J) and the five gunshots (S013-1, S035-2, S070-3, S093-4, S110-5).

	S013-1	S035-2	S070-3	S093-4	S110-5
L	15.09	15.17	15.27	14.78	15.20
C	14.20	21.64	9.20	21.02	9.42
E	16.50	13.38	18.88	19.98	18.95
H	21.80	15.06	22.12	14.22	22.03
J	20.70	21.61	16.31	16.24	16.70
σ_t	0.03	0.03	0.01	0.02	0.02

Table II: Estimated arrival times and standard errors (σ_t) [s] for the five hydrophones (L,C,E,H,J) and the two mid-frequency calls (S209-14, S210-15).

	S209-14	S210-15
L	0.00	0.00
C	-3.36	-3.38
E	-4.31	-4.29
H	5.33	5.20
J	5.94	5.92
σ_t	0.15	0.15

3. THE LOCALISATION METHOD

The following notation will be used:

- Lower case bold: column vector;
- Upper case bold: matrix;
- Lower/upper case italics: scalar

The unknown position of the source is denoted by $\mathbf{x} = (x_1,x_2,x_3)$, where the third co-ordinate indicates depth in the water. The positions of the hydrophones are denoted by $X_{i,j}$ (with i = 1,2,3 (co-ordinate index) and j = 1,2,3,4 (corresponding to the hydrophones denoted C,E,H,J, respectively)). The position of hydrophone L is taken as the origin, i.e., at (0,0,0). $\mathbf{x}$ should be solved from the following 4 equations (j = 1,2,3,4)

$$\sqrt{(x_1 - X_{1,j})^2 + (x_2 - X_{2,j})^2 + (x_3 - X_{3,j})^2} - \sqrt{x_1^2 + x_2^2 + x_3^2} = y_j = R_j - R_0 = \bar{c}(t_j - t_0) \quad (1)$$

Written shortly as

$$\mathbf{y} = F(\mathbf{x}) \tag{2}$$

where

- $\mathbf{y}$ is the measurement vector containing the ranges R_j of the source to the j^{th} hydrophone minus the range R_0 of the source to the central hydrophone L;
- $\bar{c}$ is the mean sound speed;
- t_j is the travel time of the sound from the source at $\mathbf{x}$ to the j^{th} hydrophone.

3.1. Least squares solution

Determining three unknown position co-ordinates from four relative travel times gives an inconsistent set of equations. The best estimate for the unknown co-ordinates can be found by application of least-squares adjustment. Consider a linear relation between observations (containing the relative travel times) and unknowns (the position co-ordinates)

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e} \tag{3}$$

with $\mathbf{A}$ the (m x n) design matrix, $\mathbf{y}$ the column vector of measurements (length m), $\mathbf{x}$ the column vector containing the parameters to be determined (length n) and $\mathbf{e}$ the column vector containing the measurement errors (length m). For the situation considered m = 4 and n = 3, i.e., $m > n$, an over-determined system.

The least squares solution to this problem is:

$$\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{Q}_y^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{Q}_y^{-1} \mathbf{y} \tag{4}$$

with $\mathbf{Q}_y$ the covariance matrix of the measurement vector $\mathbf{y}$. The covariance matrix of the solution reads

$$\mathbf{Q}_{\hat{\mathbf{x}}} = (\mathbf{A}^T \mathbf{Q}_y^{-1} \mathbf{A})^{-1} \tag{5}$$

which provides the precision of the solution, accounting for the uncertainties of the observations ($\mathbf{y}$) through $\mathbf{Q}_y$. The method of least-squares adjustment is based on minimizing the discrepancy between $\mathbf{y}$ and $\mathbf{A}\hat{\mathbf{x}}$.

3.2. Linearization of the problem

In our problem there is no linear relation between measurements $\mathbf{y}$ and unknowns $\mathbf{x}$. Therefore, the expression has to be linearized. A first order approximation of the nonlinear relation can be derived using a Taylor series approximation around $\mathbf{x}^{(0)}$

$$\mathbf{y} = F(\mathbf{x}^{(0)}) + \mathbf{J}\,\Delta\mathbf{x} \tag{6}$$

with $\mathbf{J}$ the (m x n) Jacobian matrix

$$J_{j,i} = \frac{\partial F_j}{\partial x_i} = \frac{x_i - X_{i,j}}{\sqrt{(x_1 - X_{1,j})^2 + (x_2 - X_{2,j})^2 + (x_3 - X_{3,j})^2}} - \frac{x_i}{\sqrt{x_1^2 + x_2^2 + x_3^2}} = \frac{x_i - X_{i,j}}{R_j} - \frac{x_i}{R_0} \tag{7}$$

evaluated at $\mathbf{x} = \mathbf{x}^{(0)}$. Further $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^{(0)}$.

3.3. The solution to the linearized problem

With $\Delta\mathbf{y} = \mathbf{y} - F(\mathbf{x}^{(0)})$ we can write

$$\Delta\mathbf{y} = \mathbf{J}\,\Delta\mathbf{x} + \mathbf{e} \tag{8}$$

To this equation we can apply the theory given in Section 3.1. The solution is

$$\Delta\hat{\mathbf{x}} = (\mathbf{J}^T \mathbf{Q}_y^{-1} \mathbf{J})^{-1} \mathbf{J}^T \mathbf{Q}_y^{-1} \Delta\mathbf{y} \tag{9}$$

$\mathbf{Q}_y$ is still the covariance matrix of the measurement vector $\mathbf{y}$, since errors on $\mathbf{y}$ are equal to those on $\Delta\mathbf{y}$.

The solution for the source position $\mathbf{x}$ is given by

$$\hat{\mathbf{x}} = \mathbf{x}^{(0)} + \Delta\hat{\mathbf{x}} \tag{10}$$

which is used in an iteration process, i.e., this solution $\hat{\mathbf{x}}$ is used as the initial value (here $\mathbf{x}^{(0)}$) for the next iteration

$$\hat{\mathbf{x}}^{(k+1)} = \hat{\mathbf{x}}^{(k)} + (\mathbf{J}^T \mathbf{Q}_y^{-1} \mathbf{J})^{-1} \mathbf{J}^T \mathbf{Q}_y^{-1} (\mathbf{y} - F(\hat{\mathbf{x}}^{(k)})) \tag{11}$$

with the Jacobian $\mathbf{J}$ evaluated at $\mathbf{x} = \hat{\mathbf{x}}^{(k)}$.

The covariance matrix of the k^{th} solution is

$$\mathbf{Q}_{\mathbf{x}^{(k)}} = (\mathbf{J}^T \mathbf{Q}_y^{-1} \mathbf{J})^{-1} \tag{12}$$

The process ends once the difference between successive solutions is negligible. For this problem typically five iterations suffice.

3.4. Calculating $\mathbf{Q}_y$

The precision of the measurements $\mathbf{y}$ is contained in the covariance matrix $\mathbf{Q}_y$ which is determined as follows. Recall that $y_j = R_j - R_0$, and denote the standard deviation of the measurement error on R_j (j = 0,1,2,3,4) as σ, i.e., $\overline{R_j^2} - \overline{R}_j^2 = \sigma^2$. Further, assume that the errors on R_j and R_k are uncorrelated, i.e.,

$$\overline{(R_j - \overline{R}_j)(R_k - \overline{R}_k)} = 0,\ j \neq k \tag{13}$$

Then it can easily be shown that the diagonal elements of $\mathbf{Q}_y$ are

$$\sigma_{y_j}{}^2 = \overline{y_j^2} - \overline{y}_j^2 = 2\sigma^2 \tag{14}$$

and the off-diagonal elements

$$\overline{(y_j - \overline{y}_j)(y_k - \overline{y}_k)} = \sigma^2 \tag{15}$$

Hence

$$\mathbf{Q}_y = \begin{pmatrix} 2\sigma^2 & \sigma^2 & \sigma^2 & \sigma^2 \\ \sigma^2 & 2\sigma^2 & \sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 & 2\sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 & \sigma^2 & 2\sigma^2 \end{pmatrix} \tag{16}$$

3.5. Properties of $\mathbf{Q}_x$

The covariance matrix $\mathbf{Q}_x$ describes the precision of the obtained position $\hat{\mathbf{x}}$ of the sound source. Sometimes only the diagonal elements of $\mathbf{Q}_x$ are considered. These diagonal elements describe the variances of the unknown parameters. However, in this way a possible correlation between the unknown co-ordinates is not accounted for. Consequently presenting errors for each coordinate separately is insufficient. The correlation between the errors in the position coordinates is relevant information and should be presented as output too. Therefore, we use the so-called confidence region, which gives the area in which the estimated position is likely to be. This confidence region is calculated from the covariance matrix. In three dimensions the confidence region in an ellipsoid. In two dimensions the confidence region is an ellipse.

Now consider the two-dimensional (2D) situation: $\mathbf{x} = (x_1,x_2)$, i.e., sound source and all five receivers are assumed to be in the same horizontal plane. The two eigenvalues of $\mathbf{Q}_x$, denoted $\lambda_{\min}$ and $\lambda_{\max}$, determine the length of the semi-axes a and b of the error ellipse according to

$$a = \sqrt{\lambda_{\max}} \quad \text{and} \quad b = \sqrt{\lambda_{\min}} \tag{17}$$

The orientation of the ellipse is given by the direction of the eigenvector corresponding to $\lambda_{\max}$. The ellipse is centered at the least-squares estimate $\hat{\mathbf{x}}$. The probability that the true position lies within the error ellipse is equal to 39 % (for the 2D situation). The 95% confidence region is obtained by multiplying the length of the semi-axes by 2.45 (assuming a Gaussian distribution).

Figure 1 illustrates for the given receiver configuration (using hydrophone L as reference) the error ellipses corresponding to a series of source positions, thereby demonstrating the localisation performance of the receiving network. For this simulation σ_t was taken as 0.1 s. Note that the size and orientation of the 95% error ellipse is determined by the receiver geometry and the source position, where source position inside the square show much smaller error ellipses than positions outside the square.

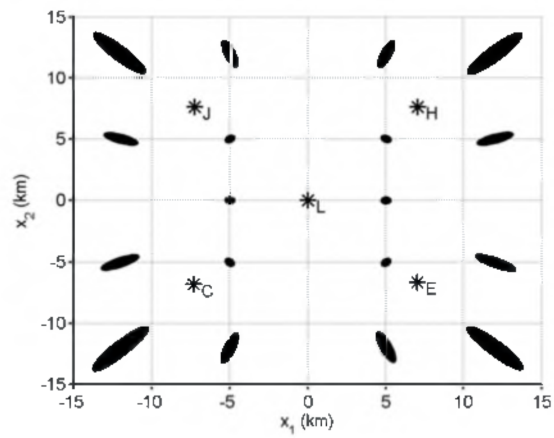

Figure 1: Simulation results: the centers of the error ellipses are the simulated source positions. The stars indicate the five hydrophone locations.

3.6. Estimating a starting solution

To assure convergence of the iterative least squares approach, an acceptable starting solution $\mathbf{x}^{(0)}$ is required. For this we assume the 2D situation with both the source and the hydrophones in the same horizontal plane.

Consider the geometry of the system of five hydrophones, forming a square with hydrophone L in its centre. Each perpendicular bisector of the line connecting two hydrophones, say J and L, defines the set of positions with equal distance to these two hydrophones, with the relative travel time to J with respect to L, t_{JL} (= $t_J - t_L$), equal to zero. All points with a positive value of t_{JL} will be at the L-side of the bisector, while a point with a negative t_{JL} will be at the J-side.

Pair-wise combination of the five hydrophones results in a subdivision of the horizontal plane in 24 sub-sectors. To prevent a too small subdivision, only 16 sub-sectors are defined as shown in Figure 2.

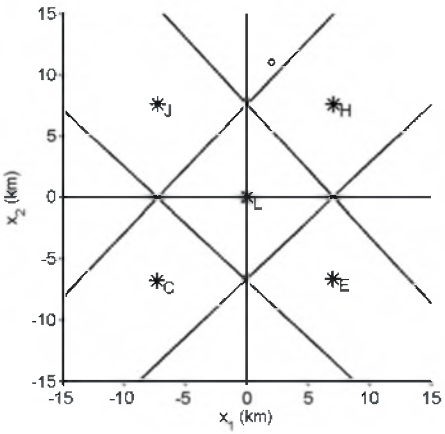

Figure 2: The 16 sub-sectors, divided by the six lines indicated. The hydrophone positions are denoted by stars, whereas the circle indicates an example initial position with $t_{CL} > t_{EL}$ and $t_{JL} < 0$.

Based on the sign of the measured relative travel times, the appropriate sub sector can be selected. As an example, the combination of $t_{CL} > t_{EL}$ and $t_{JL} < 0$ gives a position as the one indicated by the 'o' in Figure 2.

After selection of the appropriate sub sector, hyperbolic fixing is applied (Spiesberger 2001) yielding a position for each pair of relative travel times. With respect to L there are four relative travel times. With six different pair wise combinations of them and each combination leading to at most two solutions, 12 solutions result. Some of these solutions can easily be removed; the complex ones and solutions outside the selected sub sector. In this way about eight potential solutions remain. Next, ambiguous positions that are relatively far away from the main cluster of positions are removed. Taking the average of the co-ordinates of the remaining positions proves to give a good starting solution for the iterative least squares approach, wherein optimal use is made of all available travel times.

4. RESULTS

The single path assumption and the given shallow water geometry cause the relative arrival times to depend little on the source depth x_3, prohibiting determination of x_3. Henceforth we assume x_3 fixed, reducing the number of unknowns from three to two. We choose $x_3 = 0$, i.e. the sound source is at the depth of hydrophone L, but another choice would have been equally good. As a second step all hydrophones are assumed to be at the depth of hydrophone L. This assumption has proven to have a negligible effect on the estimated source position. As a consequence, the problem will be assumed 2D, i.e., $x_3 = X_{3,j} = 0$, in the remainder of this paper.

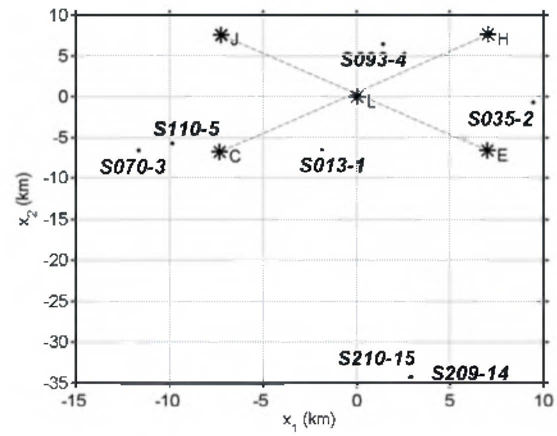

Figure 3: The 7 source positions (.) as determined by the two-stage method. The positions of the hydrophones are indicated by a star (*).

Table III: x_1 and x_2 estimates for the 7 source positions.

file	x_1 [km]	x_2 [km]	longitude	latitude
S013-1	-1.88	-6.57	-66.428	44.603
S035-2	9.45	-0.66	-66.285	44.656
S070-3	-11.66	-6.63	-66.552	44.602
S093-4	1.40	6.46	-66.387	44.720
S110-5	-9.85	-5.72	-66.529	44.611
S209-14	2.87	-34.2	-66.368	44.354
S210-15	2.95	-34.2	-66.367	44.355

The positions (x_1,x_2) corresponding to the five gunshots and the two mid-frequency calls, found with the procedure described in Section 3, are shown in Figure 3 above. The two mid-frequency calls are seen to almost coincide. Table III lists the x_1- and x_2-positions.

These estimates for the source position should not be used without an assessment of their accuracy. For example, if the uncertainty in the gunshot positions is of order several kilometers, the sound of gunshots S070-3 and S110-5 might just as well have been transmitted at the same position.

There are several contributions to the source position uncertainty (Wahlberg 2001). They stem, among other things, from uncertainty in arrival times, sound speed and hydrophone positions. These are subsequently discussed in the following sections.

4.1. Effect of uncertainty in arrival time

In Table I and II the uncertainty on the arrival times σ_t is presented. Using mean sound speed values as derived from the measured sound speed profiles, resulting uncertainties in the measurement vector **y** ($y_j = \bar{c}(t_j - t_0)$) can be estimated and the 95% confidence regions (or error ellipses), i.e., the area in which the true position is likely to fall, are calculated. The figures below show the error ellipses corresponding to the solutions presented in Table III. Figure

4 shows the results for the gunshots, whereas in Figure 5 the results for the mid-frequency calls are presented, for which the uncertainty in the arrival times is much larger than for the gunshots.

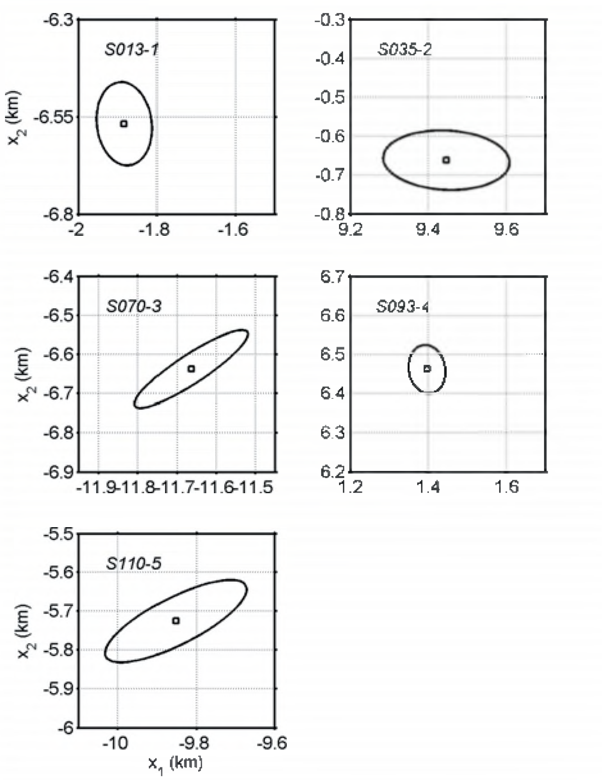

Figure 4: Error ellipses, accounting for the inaccuracies in arrival times for the gunshots. The source position estimates are indicated by squares.

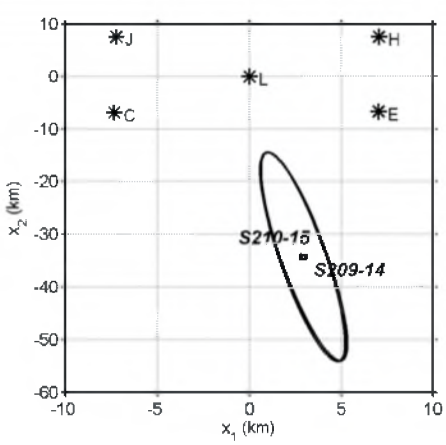

Figure 5: Error ellipses, accounting for the inaccuracies in arrival times for the mid-frequency calls. The hydrophone positions are indicated by stars. The source position estimates are indicated by squares. The positions of the two mid-frequency calls almost coincide.

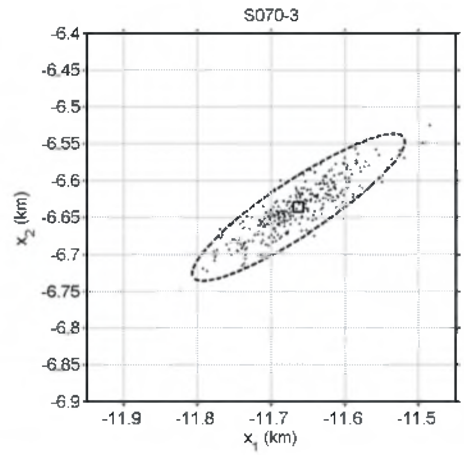

Figure 6: Results of the simulations for the gunshot S070-3. The dashed line is the theoretical error ellipse.

An alternative way for obtaining the confidence interval around the estimated source position is by means of Monte Carlo simulation. These simulations can be used for comparison with the calculated error ellipses and are also used in Section 4.3 for assessing the effects of receiver position uncertainty. The four arrival time differences are selected randomly from distributions that are assumed to be Gaussian, with means and standard deviations as given in Tables I and II. The starting position is, as previously, obtained through hyperbolic fixing. Figure 6 shows the results for gunshot S070-3. Almost all simulation results (dots) lay within the 95% confidence region as calculated according to Section 3.

4.2. Effect of uncertainty in sound speed

The effect of the uncertainty in the sound speed on the localisation accuracy is investigated as follows.

The sound speed profiles roughly show an upper layer of approximately 40 m with a sound speed of about 1499 m/s. Below the thermocline the sound speed drops to about 1489 m/s. For the calculations of Section 4.1 a typical sound speed of 1492 m/s has been used.

Here calculations are carried out for the highest and the lowest sound speed encountered, i.e., 1489 m/s and 1499 m/s. In Table IV the deviations of the thus estimated source position relative to those listed in Table III are presented. Employing these two extremes in sound speed is seen to result in deviations that are much smaller than the size of the 95% error ellipses given in Section 4.1. Hence the uncertainty in arrival times is dominating over uncertainty in sound speed.

Table IV: Shifts in estimated source position when using values for the mean sound speed of 1489 m/s and 1499 m/s, relative to the source position that is estimated using a typical sound speed of 1492 m/s.

file	1489 m/s Δx_1 [m]	1489 m/s Δx_2 [m]	1499 m/s Δx_1 [m]	1499 m/s Δx_2 [m]
S013-1	2.5	11	-5.9	-26
S035-2	-37	5.5	38	-5.6
S070-3	43	40	-95	-89
S093-4	-2.4	-8.2	5.6	19
S110-5	34	29	-79	-69
S209-14	-50	516	119	-1234
S210-15	-56	551	133	-1324

4.3. Effect of uncertainty in receiver position

Figure 7 shows the results of Monte Carlo simulations that account for the uncertainty in the receiver positions. Now the hydrophone positions $X_{i,j}$ are selected randomly from Gaussian distributions with standard deviations as given in Table V.

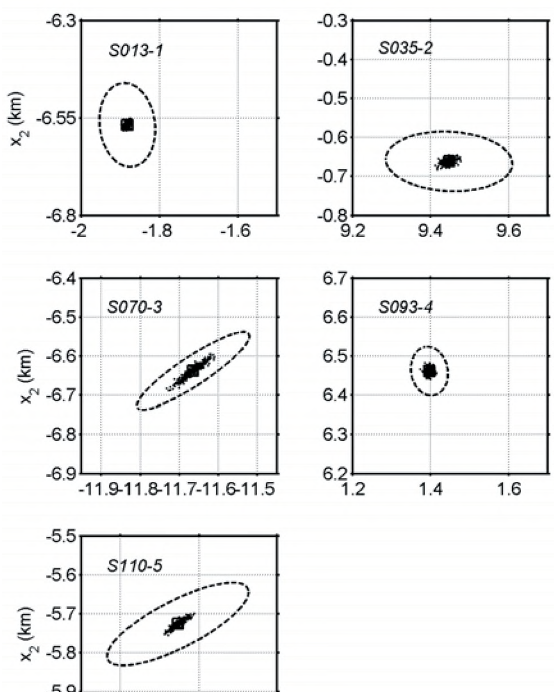

Figure 7: Results of simulations accounting for the error in the receiver positions for the five gunshots. The theoretical error ellipse, indicating the uncertainty due to uncertainty in arrival times, is plotted as a dashed line for comparison.

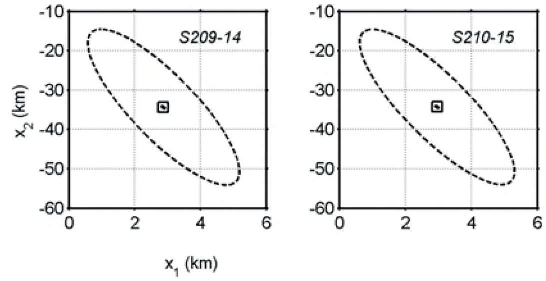

Figure 8: Results of simulations accounting for the error in the receiver positions for the two mid-frequency calls. The theoretical error ellipse, indicating the uncertainty due to uncertainty in arrival times, is plotted as a dashed line for comparison.

For comparison we have included the 95% error ellipse due to arrival time uncertainty. It is clear that the effect of hydrophone position uncertainty is small compared to the effect of the uncertainty in the arrival times.

Table V: Hydrophone position uncertainty.

Hydrophone	σ_{X_1} [m]	σ_{X_2} [m]
C	2.15	6.06
E	5.13	4.16
L	3.14	2.08
H	12.55	11.47
J	0.42	9.72

5. SUMMARY AND CONCLUSIONS

In this paper an accurate method for localizing the sound made by marine mammals is described. The method is applied to experimental data collected in the Bay of Fundy, Canada. The receiving system consists of five hydrophones, moored on the bottom. The five hydrophones form a square of 14 by 14 km with one hydrophone in the middle. Both gunshot type of signals and mid-frequency calls have been considered.

The method uses travel times of the received signals. Since the moment at which the sound was emitted is unknown, only relative travel times could be used. The middle hydrophone is taken as reference hydrophone.

The localisation procedure assumes straight path propagation in an unbounded medium and consists of two steps. The first step provides through hyperbolic fixing a first estimate for the source position, which is used as input for the second step. In this second step the solution is improved in an iterative process, where each iteration determines the least-squares solution of the set of the four linearized equations for the measured relative travel times. This so-called adjustment theory combines redundant uncertain measurements in an optimal way, by weighting the observations with a measure of confidence and by determining the least squares solution. The method also gives the precision of the estimated source positions by presenting the 95% confidence region giving the area in which the estimated position is likely to be.

The gunshot signals have been accurately localized, whereas the mid-frequency calls show a much larger position uncertainty due to the larger uncertainty in relative travel times and the larger distance to the hydrophone array. The precision of the gunshot positions, i.e., the size of the error ellipse, is of order 200 m, whereas it amounts to several tens of kilometers for the mid-frequency calls. It should be emphasized that the correlation between the errors in the position co-ordinates is important and is therefore presented too by the orientation of the error ellipse.

An important item addressed in this paper is an assessment of the position accuracy due to uncertainties in arrival times, sound speed and receiver position. It is found that for the experiment considered, the uncertainty in the arrival times is dominant.

ACKNOWLEDGEMENT

The authors are grateful to the workshop organizers for making available the data set. Wilco Boek (Underwater Acoustics Group) and Huib de Ligt (Mathematical Geodesy and Positioning) are acknowledged for their help and valuable advice.

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