Comparing A Linear With A Non-Linear Method For Acoustic Localization

Magnus Wahlberg
Department of Zoophysiology, Aarhus University, C. F. Moellers Alle Building 131, DK-8000 Aarhus C, Denmark

ABSTRACT

The performance of two different acoustic localization techniques is evaluated with signals from right whales in the Bay of Fundy. The methods are compared to the GPS localization error (114-273 m, N=3) through the use of played back whale calls. The linear approach underestimates the source location error (22 m, N=3), whereas the non-linear approach exaggerates the error (462-1166 m, N=3). The linear approach may render unrealistic error bounds because of the inherent non-linear properties of the localization problem. The non-linear approach may exaggerate error bounds by choosing the wrong cross-correlation peak for the time-of-arrival difference measurements. Whereas the GPS localization error was always contained within the non-linear error bounds it was never contained within the linear error localization bounds. This indicates that the non-linear approach can give more realistic error estimates, especially in situations where the sound path geometry is unknown. [Work supported by the Office of Naval Research and the Oticon Foundation.]

SOMMAIRE

La performance de deux méthodes différentes de localisation acoustique est évaluée à partir de la localisation acoustique des baleines franches dans la Baie de Fundy. Les méthodes sont comparées à l'erreur de localisation GPS (114-273 m) à partir de vocalisations de baleines franches préenregistrées. L'approche linéaire sous-estime l'erreur de localisation de la source sonore (22 m), alors que l'approche non-linéaire surestime l'erreur (462-1166 m). L'approche linéaire rend irréaliste la marge d'erreur possible à cause des propriétés non-linéaires du problème de localisation. L'approche non linéaire exagère la marge d'erreur, ce qui est expliqué par le choix du mauvais maximum de corrélation croisée des mesures de différences de temps d'arrivée. Toutefois, l'erreur de localisation GPS était toujours contenue à l'intérieur d'une marge d'erreur non-linéaire et n'était jamais contenue à l'intérieur d'une marge d'erreur linéaire de localisation. Ceci indique que l'approche non-linéaire peut donner des erreurs d'estimation plus justes, spécifiquement dans les situations où la trajectoire du son est inconnue. [Travail supporté par l'Office of Naval Research et la Oticon Foundation.]

1. INTRODUCTION

In bioacoustics it is often relevant to determine the location of a calling animal. In studies ranging from acoustic census of animal populations to behavioural studies the knowledge of animal location greatly extends the types of problems that can be addressed and broadens the analytical techniques that can be applied.

Acoustic localization is performed using a receiver array to locate a vociferous animal by measuring the arrival times of corresponding signals at different receivers (Wahlberg *et al.* 2001; Spiesberger and Fristrup 1990). Methods for acoustic localization are currently in rapid development, in terms of both recording and analysis techniques (see Møhl *et al.* 2001 and the papers in the present volume of this journal).

How many receivers are needed for specific localization tasks, assuming all receivers to have omni-directional receiving characteristics? We may view acoustic localization as a mathematical transformation from measured time-of-arrival differences (TOADs) to the source coordinates. With an array of N receivers one can measure $N-1$ independent TOADs. (The word *independent* is here used with the meaning of two variables not being 100% correlated. The $N-1$ TOADs mentioned in the text which are not linear combinations of one another, are not completely uncorrelated with each other. E.g. the TOAD between the signal at receiver 1 and 2 and the TOAD between receiver 1 and 3 both contain a measurement of the time-of-arrival at receiver 1. See Wahlberg et al. (2001) for details.) By convenience the TOADs are measured between each receiver and one reference receiver, denoted receiver 1, defined as being in the origin of the coordinate system. Any other definable TOAD from an N-receiver array could be expressed as a linear combination of the other TOADs: for example, the TOAD measured between receiver 2 and 3 is the same as the TOAD between receiver 3 and 1 minus the TOAD between receiver 2 and 1.

From mathematical analysis we know that $N-1$ TOADs may be transformed into a maximum of $N-1$ source coordinates. To track an animal in two dimensions two source coordinates are required. To achieve this one needs to measure at least two independent TOADs, and the array must consist of at least three receivers. For three-dimensional tracking requiring three coordinates a minimal number of four receivers is needed. Following the notation of Wahlberg *et al.* (2001), we call such an array a *Minimum receiver number array* (MINNA). Arrays with more receivers than MINNAs are called ODAs (*over-determined arrays*; Wahlberg *et al.* 2001). For ODAs more TOADs are available than minimally required for calculating the source coordinates. In this case the source coordinates can be calculated through some kind of averaging technique, such as least squares.

The number of TOADs necessary to solve a certain localization task may also be determined from geometric considerations. Each TOAD restricts the source location either to a hyperbolic curve (in a 2-D source-array geometry) or to a hyperboloid surface (in 3-D). Two intersecting curves (corresponding to two TOADs) are sufficient for localizing animals in 2-D (Fig. 1). In 3-D, three intersecting surfaces are needed to localize the animal.

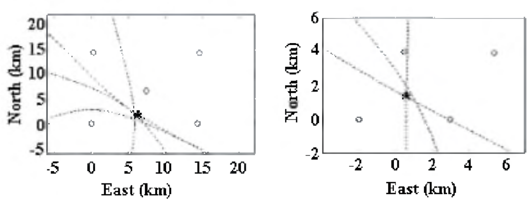

Figure 1. Hyperbola plot of (left) a right whale vocalization recorded in 2002 (file nr. S093-9) and (right) a played back right whale call recorded in 2000 (file no. S282). The circles indicate receiver locations. The least-squares estimate of the source location is denoted with an asterisk ('*').

The simplest acoustic location equations usable for MINNAs are quadratic (Wahlberg *et al.* 2001), potentially rendering two source solutions instead of one for each set of TOADs. Geometrically this corresponds to two hyperbolas or three hyperboloids intersecting in two rather than only in one point. In such cases an extra receiver is needed to resolve the source location ambiguity (Spiesberger 2001). The introduction of an extra receiver renders an extra TOAD and therefore an extra hyperbola/ hyperboloid. This array is now an ODA.

Assuming that the sound speed of the medium is constant, the source location can be derived from MINNAs using analytical equations. These algorithms are invertible, except for locations where source locations are ambiguous. A problem with MINNA arrays is that there is no implicit information available on the accuracy of source coordinate estimates. The investigator has to rely upon error propagation analysis to evaluate the magnitude of localization errors (Wahlberg *et al.* 2001). For ODA systems, the redundant TOADs may be used to either assess the error in source location through regression techniques, or through an analysis of error propagation (Spiesberger and Fristrup 1990; Wahlberg *et al.* 2001). The transformation from TOADs to source locations is in general not invertible for ODA systems, as the calculation involves some type of data smoothing.

Both for MINNA and ODA analysis, the simplest form of an error analysis is to linearize the location problem and its error components (Spiesberger and Fristrup 1990; Wahlberg *et al.* 2001). Linearization has the advantages of yielding fast computations with well-defined procedures for the error estimation. The magnitude of various error sources, such as variations in the sound speed, inaccuracies in TOAD measurements, and drifting receiver locations, can be studied and modelled separately. Also, one may rapidly evaluate how any covariance between the input variables affects derived source locations (Wahlberg *et al.* 2001).

However, linearizing the localization equations introduces several problems. If the hyperbolas are not crossing, there will be no source location (The source location coordinates will in this case be a complex number) in a MINNA system, even if the shape of the hyperbolas indicates that the source ought to be restricted to a certain area. Even if a source location is obtained, there is no possibility of verifying that the input TOADs were measured to a stated accuracy. These problems are alleviated through the introduction of another receiver. However, it should be recalled that the localization equations of both MINNAs and ODAs are inherently non-linear, and the degree of non-linearity is spatially variant within and around the array. Therefore, linearization may work acceptably in some cases but not in others (Spiesberger and Wahlberg 2002).

Such problems call for the development of non-linear localization methods. Spiesberger and Wahlberg (2002) developed a non-linear form of acoustic location error analysis based on computer simulations of permuted subsets of MINNA receiver constellations (Fig. 2). Using synthesized data, this numerical form of error analysis seemed to give more realistic error estimates than linear analysis. The authors noted the need for application to real data before the method's performance could be fully evaluated (Spiesberger and Wahlberg 2002).

In November 2003 a workshop was organized in Halifax on passive acoustic localization of marine mammals (Anon. 2003). Before the workshop, the organizers supplied the participants with right whale *(Eubalena glacialis)* recordings as a training dataset for investigating alternative localization and detection routines. The dataset also included playback recordings, where signals were broadcasted from known locations. The dataset provided an opportunity for comparing the performance of the linear and the non-linear localization methods outlined above.

2. METHODS

2.1 Data material

Sound recordings were obtained from the organizers of *the Workshop on detection and localization of marine mammals using passive acoustics, 19. – 21. Nov. 2003* (Anon. 2003). Data was collected with 5 Ocean Bottom Hydrophones (OBHs, *Defense Research & Development*

Canada, Halifax) moored in the Bay of Fundy in September 2002 in an area where foraging right whales (*Eubalaena glacialis*) are regularly observed during summertime. Recordings were made with a sampling rate of 1200 Hz. The dataset is described in detail in Anon. (2003). The bottom depth varied between 123-210 m at the site of the hydrophones. The locations of the receivers were determined both by GPS and by recording playback signals at known locations. The total error in receiver coordinates was acoustically assessed to vary between 4 and 18 m. TOAD measurements were prone to errors arising from differential clock drift in the OBH recording units (Anon. 2003). The clock drift was measured both before and after the fieldwork and varied between 65 and 174 μs per hour. Signal time-of-arrivals at each OBH recording were compensated assuming that the clock drift was linear throughout the recording period. The workshop organizers supplied 16 sound files containing right whale vocalizations, which had been classified as being either 'low-frequency', 'mid-frequency', or 'gunshot' calls (Anon. 2003). In addition, sound speed data was derived from 6 conductivity-temperature-depth (CTD) profiles measured closely in time to the sound recordings.

From data gathered during a previous array deployment in August 2000 in the same area the performance of the location system was assessed. Right whale sounds were transmitted from a small boat at a known location (Anon. 2003). This data was used to investigate the precision of acoustic location and anticipated error estimates. During these recordings, the array consisted of four (instead of five) OBH's, moored at 131-190 m depth, the source being placed at 20 m depth. The workshop organizers supplied four sound files from the playback sessions. In addition, data was made available from 3 CTD profiles obtained in the area at the time of the recordings. There was no acoustic calibration of the receiver locations during the 2000 recordings. The field recordings were made during such a short time interval that compensation for the buoy clock drift (see above) was considered unnecessary (Anon. 2003).

2.2 Analysis

Data were extracted with sound-analysis software (*Cool Edit*, Syntrillium), and measurements were made with scripts written in *Matlab 6.5* (Mathworks, Inc.). TOADs were measured by cross-correlating signals recorded at different receivers. The TOAD measurements included compensation for the buoy clock drift in the right whale recordings (see above), assuming the clock drift rate to be constant during the recordings. The TOADs, the sound speed, and the receiver locations were used to calculate the location of the source as well as the associated error (*sensu* Wahlberg *et al.* 2001 and Spiesberger and Wahlberg 2002). A linear and a non-linear method method were compared in the localization process.

The vertical aperture of the array (the differences in bottom depths between the receivers) was much smaller than (less than 1%) the horizontal distance between the receivers. Thus, the array is situated approximately in the horizontal plane. The water depth was considered insignificant (about 1-2 %) compared to the horizontal extent of the array. It was not expected to be possible to locate sound sources in the vertical plane with a resolution better than the depth of the water column. Therefore acoustic location was made with 2-D versions of the algorithms presented in Wahlberg *et al.* (2001) and Spiesberger and Wahlberg (2002). These algorithms assume both the source and the receiver array being situated in the same horizontal plane.

In the *linear error estimation* approach, the source location was assessed through the least squares technique described by Spiesberger and Fristrup (1991) and Wahlberg *et al.* (2001). A homogenous sound speed was assumed. Error estimates of the source location coordinates were achieved using a linear error propagation model (LEP, Wahlberg *et al.* 2001) applied to the input variables. As input to the calculations, the input variables and their errors were required, as well as the covariances between the variables (Wahlberg *et al.* 2001).

The *non-linear error estimation* approach is described in detail in Spiesberger and Wahlberg (2002) and is depicted in Fig. 2. The array is divided into a number of MINNA subunits, each containing 3 or 4 receivers (for 2-D and 3-D positioning, respectively). Each subunit has a set of input variables: the receiver locations, the TOADs, the sound speed and error estimates. The corresponding source location is calculated through the MINNA localization formula given in Wahlberg *et al.* (2001). If the sound speed varies between the source and the different receivers, a set of quadratic equations, called *isodiachron equations* (Spiesberger and Wahlberg 2002) can be used alternatively. Therefore the non-linear analysis is not restricted to the assumption of a homogenous sound speed. The input variables and their estimated errors are used to randomly shift the sound speed, receiver locations, and TOADs of the MINNA system. For each shift in input variables a new source position is derived. We assessed 1000 locations for each MINNA sub-array. This generates a cloud of possible source locations. For any TOAD that generates a doublet location an additional receiver is chosen to solve the ambiguity. The procedure is repeated for all the MINNA sub-array constellations, or (if there are too many constellations) for a Monte Carlo Subset of these constellations. Each constellation generates a cloud of possible source locations. The location of the source is defined as being the surface (in 2-D) or volume (in 3-D) where all the generated clouds intersect. This intersection is the only possible region in space where the source must be situated, provided the input variables are given with adequate error intervals and the sound speed is constant between the source and each receiver (see Spiesberger and Wahlberg 2002 for details).

2.3 Error assessment

The error analysis of both the linear and non-linear approach demands proper assessment of the accuracy of all input variables. The input variables are the sound speed, the TOADs, and the receiver coordinates. The error assessment is considered in some detail for the data used for the workshop (Anon. 2003).

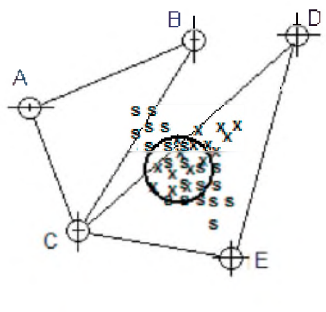

Figure 2. The principle of the non-linear method for acoustic localization, adapted after Spiesberger and Wahlberg (2002). Circles are receiver locations with error bars. Two subset MINNA receiver constellations are shown, ABC and CDE. The source locations derived from varying the errors in the input variables of the ABC subset are denoted with 's', and the corresponding source locations from the CDE subset are denoted with 'x'. The source is defined as being within the region enclosed by the source solution of the two subsets, marked with a circle.

Sound speed

The linear analysis assumes that the medium has a constant sound speed. The non-linear approach allows the sound speed to vary with the direction of source to receiver, but not with range. In the real ocean sound speed normally varies both horizontally and vertically, the most pronounced gradients usually being vertical. Propagating sound waves are refracted in a sound speed gradient according to Snell's law (Urick 1983). The acoustic path from the source to the receiver may pass through a range of sound speeds. Consequent ray bending can be studied using ray-tracing. If detailed algorithms are not applied, Spiesberger and Fristrup (1990) have derived an alternative approximation to quantify the effect of ray bending on TOAD measurements.

In the present study, sound speed profiles were averaged both spatially (vertically) and temporally, and the standard deviation calculated. The mean and the magnitude of one standard deviation were used as inputs to the error assessment of source localization. In 2002 the sound speed was 1494 ± 3 m/s; in 2000 it was 1494 ± 6 m/s.

Time-of Arrival Differences (TOADs)

There are many available methods for measuring TOADs for corresponding signals recorded on different receiver channels. The most widely used technique is cross-correlation. In general cross-correlation performs well if the signal has a large time-bandwidth product (Spiesberger and Fristrup 1990). The width of the peak of the cross-correlation envelope function is given by Spiesberger and Fristrup (1990):

$$\delta t \approx 1 / [\, 2\pi\, W_{rms}\, d \,]$$

δt is defined as the time-of-arrival measurement inaccuracy, W_{rms} is the rms bandwidth of the signal, while d is the linear signal-to-noise ratio of the cross correlator. The cross correlator signal-to-noise ratio can be derived from the signal-to-noise ratio of the recordings (defined as the rms intensity of the right whale signals divided by the rms intensity of the noise in the frequency band of the signal), which was measured to be 13-22 dB, and from the number of samples in the digitized signal (see Spiesberger and Fristrup 1990). The number of samples in each right whale signal is about 1500-2000. The TOAD of right whale calls (having a rms bandwidth of 9-10 Hz) is estimated to be measured with an accuracy δt of about 4 to 29 µs. A value of 30 µs was used in the error assessment of acoustic localization presented below. The sampling frequency of 1200 Hz corresponds to a sample time resolution of 833 µs, which is larger than the 30 µs time resolution. Therefore, the cross correlation function was interpolated ten times to resolve arrival times at a scale dictated by the calculated timing accuracy. A typical cross correlation from the playback localizations is shown in Fig. 3. There is one well-defined peak of the cross correlation function, but also there is a whole series of peaks, probably caused by multiple paths from the source to the receiver. As will be discussed below, the precision of 30 µs is only valid if we assume that the correct cross-correlation peak has actually been measured. (An even better TOAD resolution may be obtained from using the peak of the cross correlation function, rather than the peak of the envelope of the cross correlation function. However, for the present cross correlation signals it was often difficult to assess which peak in the cross-correlation function should be chosen, whereas the envelope function usually rendered an unambiguous peak (c.f. Fig. 3).]

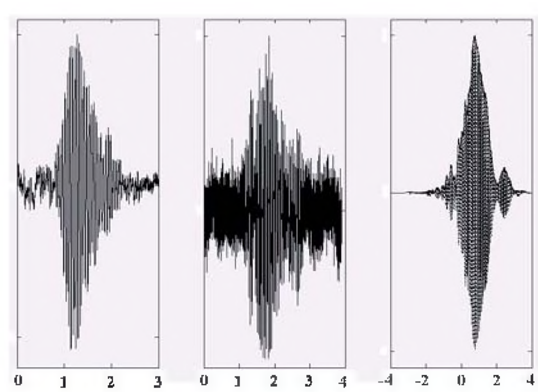

Figure 3. The cross correlation of a call recorded on two of the receivers of the array during 2003. Left: 'Gunshot call' recorded on the buoy C, Middle: the same 'gunshot call' recorded on buoy H. Right: The cross-correlation (stipled) and its envelope (solid line) of the two signals in (a) and (b).

Receiver coordinate errors

Receiver coordinate errors were assessed using the pinger recordings in 2002. The estimated error in the north-south direction was 2-12 m, and in the east-west direction it was 0.5-13 m (Anon. 2003). For the playback recordings from 2000 no measurements of receiver coordinate errors were available. For the analysis of the playback signals it was assumed that the north and east mean values in receiver location errors were equal in 2002 and 2000. This assumption may be too liberal but was used due to the lack of better data.

3. RESULTS

3.1 Error maps

Once the magnitude of the errors in the input variables have been defined, the LEP model may be used to derive error contours for the array. If it is assumed that the errors in sound speed, TOADs and receiver coordinates are uncorrelated, then the error maps can be split into the contributions from each error source (Wahlberg *et al.* 2001). This procedure is useful for evaluating the localization precision of various source-to-array geometries, and also to pinpoint which input variable errors have the largest effect upon the localization error. Fig. 4 shows an example of error maps so derived. The source location error seems mainly to be caused by errors in sound speed and receiver locations, rather than in the precision of the TOAD measurements.

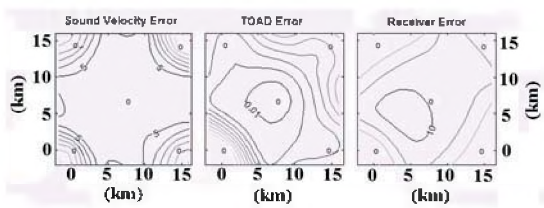

Figure 4. Error maps of the array used in 2002, calculated with a linear error propagation model (see text for details). Receiver locations are indicated with circles. The effect on the localization error is shown separately for the sound speed error (left, 5 m error contours, dc= 3 m/s), the TOAD errors (middle, 1 cm error contours, dt=30 µs), and errors in receiver coordinates (right, 10 m error contours, receiver errors from Anon. 2003).

3.2 Choices of error distributions and error estimates for source coordinates

The assessed variables and their errors are fed into the linear and non-linear acoustic localization analyses. For the linear approach we may choose between studying the residual error of the least-squares fitting, or to use the linear error propagation model to assess the magnitude of the errors. With all other quantities held constant these two error calculations should render comparable results. If not the case, it is usually a sign of a problematic localization task, e.g. that one of the TOADs has been erroneously interpreted and measured.

For the non-linear approach it is necessary to define the shape of the error distribution about the variable's mean value. When performing error analysis it is usually assumed that the errors are normally distributed and can be modelled as measured in terms of standard deviations or standard errors. However, the normal distribution may not be the best way to define all error limits. For example, the tails of the normal distribution does not fall to exactly zero. Therefore, if one assumes normally distributed receiver locations there is always a small chance that the receiver is at an arbitrarily large range from the other receivers, which for physical reasons cannot be true. A better approach is to use truncated normal distributions, uniform distributions or other distributions with well-defined limits. In the work presented here we choose a uniform distribution for the nonlinear analysis. This renders comparable results for the linear model if we assume that the standard deviations of the linear model represents the range of a uniform distribution rather than a normal one. It is believed that the discrepancy between the two error distributions is minor and does not significantly influence the comparison between the linear and non-linear error estimations.

3.3 Acoustic localization of playback signals

Three out of the four playback files from 2000 contained events where the sound source could be located. The dropped file (S-289) was apparently dominated by source multi-path making definite cross correlations impossible.

In Table 1 the discrepancies between logged position of the play back vessel and the acoustic location of the signal are compared to the linear and non-linear source location error estimates. Fig. 1 (right) shows a sample hyperbola plot from the acoustic localization of a play back signal.

Table 1. The error in acoustic localization of play back signals ('ODA-GPS') compared to the localization error assessed with the linear ('ODA-LEP') and the non-linear techniques (see text). The linear and non-linear errors are given in meters, and in % relative the ODA-GPS error.

Seq.	Difference ODA - GPS	ODA LEP	Non-linear Error
S-282A	114 m	22 m (19%)	1166 (1023%)
S-282B	273 m	22 m (8%)	462 (169%)
S-288	174 m	22 m(13%)	1151 (661%)

3.4 Acoustic localization of right whale calls

Four out of 16 files from 2002 contained right whale signals which yielded cross-correlation functions usable for sound localization. The remaining files were problematic, either due to signal overload (the signals were digitally clipped in 4 of the files), interference between calls from several whales (in 4 files), or poor signal-to-noise ratio (in the remaining 4 files). An example of a successful acoustic location is depicted in Fig. 5. The mean and 1 s.d. error estimates from the linear error propagation model are depicted as a black cross. The corresponding nonlinear error limits are depicted as a rectangle. In Fig 5 (top) a 30 µs TOAD error estimate is used.

The linear error in Fig. 5 (top) is so small so that the error cross cannot be observed on th emap. To evaluate the risk of choosing the wrong cross-correlation peak, in Fig. 5 (bottom) the TOAD error is increased to 1 second which approximates the maximum distance between the peaks in the cross-correlation envelope function.

4. DISCUSSION

This first trial compares the performance of the linear and non-linear error methods from Wahlberg *et al.* (2001) and Spiesberger and Wahlberg (2002) using real data. The linear error model underestimates errors whereas the non-linear model over-estimates them (Table 1). The GPS localization error (defined as the difference between the GPS position - with a 10 m error of its own – and the ODA acoustic location) is never contained within the linear error estimates, whereas it is always contained within the non-linear error estimates (Table 1). One may therefore claim that the non-linear approach renders the most realistic error estimates.

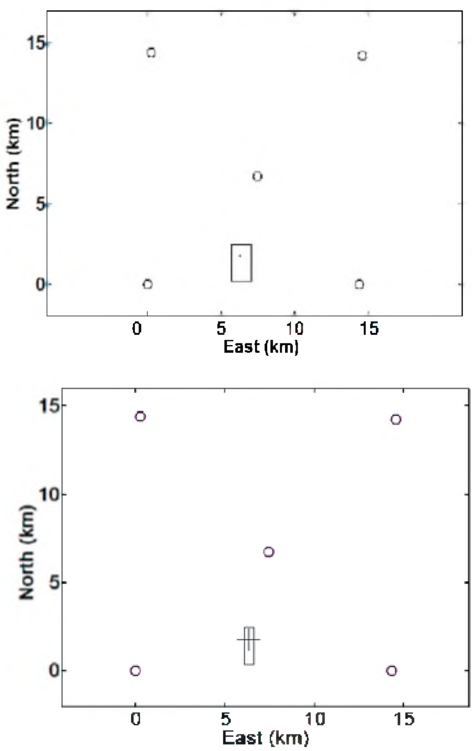

Figure 5. Acoustic localization of right whale number S093-09 (from Anon. 2003). The localization error estimated from linear error propagation is shown as a cross (in the top figure the cross is so small that it looks like a dot). The error estimated from the non-linear approach is depicted as a rectangle. The TOAD error is set to 30 µs in top, and to 1 s in bottom. See text for details.

A problem with the linear model is that the both the localization and its associated error analysis is achieved with linearizing techniques, despite the fact that the acoustic localization problem is non-linear in nature. This problem compounds by additional unrealistic approximations: the sound speed is set constant, and one assumes that the correct cross-correlation peak is chosen in the presence of multi-path. The huge discrepancy between the linear and non-linear error estimates (Fig. 5 top) assuming a TOAD uncertainity of 30 µs is alleviated by increasing the TOAD uncertainty to 1 s (Fig. 5 bottom). This latter TOAD error better approximates reality considering that one may select the incorrect cross-correlation peak when measuring the time-of-arrival differences.

The non-linear approach yields more realistic error estimates as it presents the true range of possible source locations, given correct input data. This approach, while not requiring any linear assumptions, still assumes that the sound speed is constant (even though the algorithm may be modified to contain cases where the sound speed is variable between different source – receiver paths, see Spiesberger and Wahlberg 2002). Although the true source location error is always found within the non-linear error estimates (Table 1) the non-linear errors frequently appear to be almost an order of magnitude too large. The reason for this is not clear. It may indicate that the model is wrong: we may not have chosen the right cross-correlation peak, the sound speed profile causes ray bending so that we are not detecting the direct path but surface and bottom reflections, etc. All these reasons should have affected the derived source location. However it is not clear why they only affect the non-linear error estimation.

The non-linear approach has the advantage of treating each array as a constellation of several MINNA arrays. Each MINNA location is a reversible transformation from the input variables to the source coordinates (with the removal of ambiguous locations using an additional receiver; see above). Therefore, variations in the input variables within the assessed error limits are directly transformed into variations in the source coordinates that are reversibly related to the original input variables. In other words, for each MINNA system, the source location has to be exactly where it is calculated to be. Therefore the cloud of locations derived from each MINNA system directly reflects the only possible limits of the source location coordinates. Furthermore, the input variables may have different error distributions, and such effects can be directly observed upon the shape of the location cloud. When combining all the MINNA subsystems comprising the array, and always assuming that the input variables are accurately describing the real recording situation, the source location must lie inside the space defined by the intersection of all the location clouds.

Therefore, while the linear approach gives a possibly faster approximate source location, the non-linear approach inherently generates error bounds that reflect the only possible location of the source, given that the input variables and their error ranges are realistic.

The problem of computing time for numerical models has diminished with faster computers. The non-linear calculations made here can be accomplished on a standard laptop within a few seconds. For larger array systems, longer calculation times are expected, so the feasibility of the non-linear method decreases - especially for online applications.

Acoustic localization is a non-linear acoustic problem that can be solved either through linearization, or through non-linear techniques. The inherently non-linear nature of the localization problem suggests that only the non-linear approach can be used for more sophisticated future models. Such non-linear techniques could also be a gateway towards inverse acoustic localization methods, such as matched-field and inverse processing (Thode *et al.* 2000; Spiesberger 1999).

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