ACCURACY IN THE LOCALIZATION OF SPERM WHALES RESIDENT IN THE STRAIT OF GIBRALTAR USING ONE HYDROPHONE

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ABSTRACT

The geographical distribution in the Strait of Gibraltar of sperm whales (*Physeter macrocephalus*) and of four species of dolphins suggests some common foraging territories between the species, and a subsequent share of the water column. A localization method using one hydrophone at an unknown depth has been used here to estimate the foraging depth of sperm whales. The sperm whales tracked in the Main Channel of the Strait of Gibraltar have been found to be hunting at the bottom of the water column.
The localization method calls for the use of a large vertical 4-hydrophone array. The accuracy of this method is assessed using random variables. A simple analytic expression the error on the depth of the source is then calculated. The results of the localization method are checked by considering an second sperm whale and a second hydrophone.
Three ray propagation models have been compared to study the importance of the heterogeneity of the speed of sound in the localization process.

RÉSUMÉ

Des études visuelles en surface dans le Détroit de Gibraltar montrent que les grands cachalots (*Physeter macrocephalus*) et plusieurs espèces de dauphins partagent leurs territoires de chasse. Une méthode de localisation par acoustique passive utilisant un hydrophone à une profondeur inconnue a été utilisée ici, afin d'estimer la profondeur de chasse des cachalots, et prouver un éventuel partage de la colonne d'eau entre les différentes espèces. Les cachalots poursuivis dans le Détroit de Gibraltar chassent dans la partie inférieure de la colonne d'eau.
La méthode de localisation simule l'utilisation d'un réseau vertical à 4 hydrophones de grande dimension. La précision dans la méthode de localisation est estimée à l'aide de variables aléatoires. Une expression analytique de l'erreur sur la profondeur de la source est donnée. Un second cachalot et un second hydrophone ont été considérés afin de vérifier les résultats de la méthode de localisation. Trois modèles de propagation ont été comparés afin d'étudier l'importance de l'hétérogénéïté de la célérité du son dans l'estimation de la position de la source.

1 INTRODUCTION

The Strait of Gibraltar, situated between western Europe and northern Africa, is a common place for many species of toothed whales. Visual countings from the sea surface enable a plotting of the geographical distribution of the whales, and show that some species do share common surface territories, especially long finned pilot whales (*Globicephala melas*) and sperm whales (*Physeter macrocephalus*). The vertical repartition of these two species during hunting, and likely a clear share of the water column, can be assessed by locating the animals using passive acoustics. The localization method using one hydrophone at an unknown depth [Thode et al., 2002] has been applied, as it is cheap and easy to set *in situ*.

The localization method may be very sensitive to errors in the measured data, and its accuracy is estimated by

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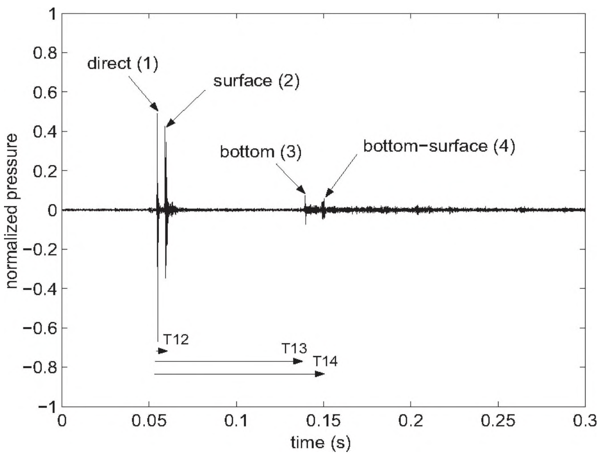

Figure 1 - **Received signal of a click.** It is composed of the click transmitted *via* the direct path (1), reflected once by the sea surface (2), reflected once by the sea bottom (3), and reflected twice at first by the sea bottom then by the sea surface (4).

using random variables. The theory of this uncertainty estimation technique is developed here in general terms, and can also be applied in other localization methods.

A constant speed of sound along the water column has been at first applied when locating. But variations in speed of sound may be important, and the consequences of these variations on the measurement of the data are investigated. The previous localization method is then enhanced, to take into account these variations in speed of sound.

2 CLICKS AND ECHOES

The oceanic propagation field is limited by two interfaces, the sea bottom and the sea surface, which diffuse and reflect acoustic waves. The signal emitted from a source travels *via* different paths, and arrives at a receptor as the sum of the signals transmitted *via* a direct path and its multireflected echoes. The shallow basin of the Strait of Gibraltar allows such multipath propagation to take place. Surface and bottom echoes of sperm whale clicks are commonly detected in the Strait of Gibraltar (Figure 1).

The sperm whale signal received by the hydrophone, as the sum of the sperm whale clicks transmitted *via* the direct path and its three delayed echoes, can be interpreted as the sum of four signals received on four hydrophones (the real one and three additional virtual ones). Each sperm whale click is then recorded on a large vertical

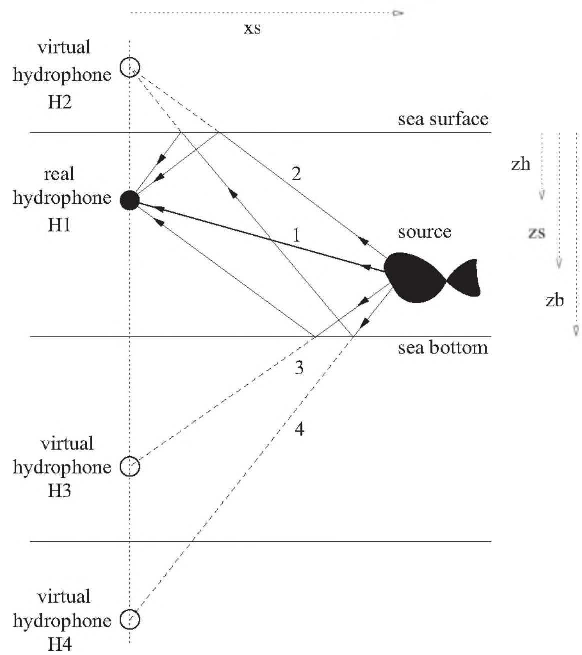

Figure 2 - **Virtual large vertical 4-hydrophone array.** The depth z_h of the hydrophone, the depth z_s of the source, and the range x_s of the source from the hydrophone are estimated from the measurement of the depth z_b of the sea bottom, the speed of sound, and the delays between the four received signals.

4-hydrophone array (Figure 2).

3 CONSTANT SPEED OF SOUND LOCALIZATION

By measuring delays, a source can be localized in range and in depth by using three hydrophones. These hydrophones are to be located on a common vertical line at known depths. Each sperm whale click is recorded here on a virtual 4-hydrophone array. The depth z_s of the sperm whale can be found by using any of the four different triplets of hydrophones ($\{H_1, H_2, H_3\}$, $\{H_1, H_2, H_4\}$,$\{H_1, H_3, H_4\}$, or $\{H_2, H_3, H_4\}$), as a function of the depth z_h of the hydrophone H_1. Considering the 3-hydrophone array $\{H_1, H_2, H_3\}$, the depth of the source is :

$$z_s = \frac{4z_b(z_b - z_h)\tau_{12} - c_0^2\tau_{12}\tau_{13}\tau_{23}}{4z_b\tau_{12} + 4z_h\tau_{23}} \quad (1)$$

and by considering the array $\{H_1, H_3, H_4\}$, it is found as :

$$z_s = \frac{4z_b(z_b + z_h)\tau_{13} - 4z_b(z_b - z_h)\tau_{14} - c_0^2\tau_{13}\tau_{14}\tau_{34}}{-4z_b\tau_{34} + 4z_h\tau_{14}} \quad (2)$$

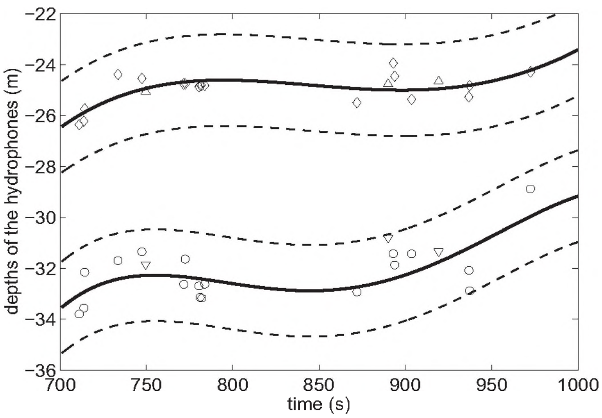

Figure 3 - **Depths of the hydrophones.** The depths of the hydrophones are found from the delays on clicks emitted from the first sperm whale (◯ and ◇) and from the second sperm whale (△ and ▽). The results are interpolated, and a 95 % uncertainty band is plotted, of width $4\sigma_{z_h}$ ($\sigma_{z_h} = 0.9$ m).

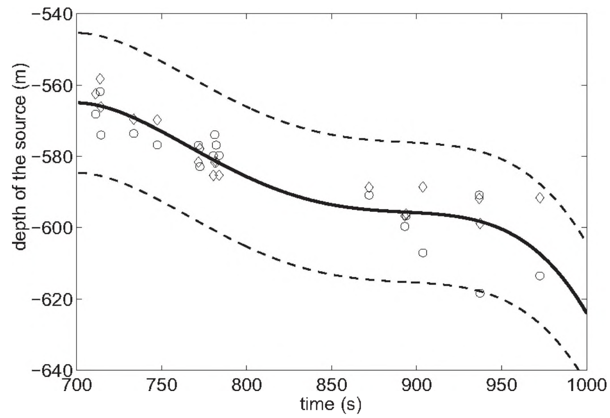

Figure 4 - **Depth of the sperm whale.** Depth of the sperm whale found from clicks measured on the first hydrophone (◯) and on the second hydrophone (◇). A 95 % uncertainty band is plotted, of width $4\sigma_{z_s}$ ($\sigma_{z_s} = 9.8$ m).

The depth z_s of the source is a function of the following parameters, the depth z_b of the sea bottom, the speed of sound c_0 (a supposed constant along the water column), and the delays τ_{1i} of the signal transmitting *via* the path i on the signal transmitting *via* the direct path (Figure 1). The time differences $\tau_{ji} = \tau_{1i} - \tau_{1j}$ represent the differences between the arrival times of the signals following path i and path j or, equivalently, arriving at hydrophone H_i and hydrophone H_j.

The depth of the source also depends on the depth z_h of the hydrophone. A unique solution for the depths z_s of the source and the depth z_h of the hydrophone can then be found from (1) and (2) given some asumptions on the depth of the source ($z_b - z_h < z_s < z_h$).

The virtual 4-hydrophone array considered here is not overdetermined, as the depth of the real hydrophone H_1 is unknown. Only two 3-hydrophone arrays ($\{H_1, H_2, H_3\}$ and $\{H_1, H_3, H_4\}$) have been used in the localization process. The use of additional arrays (such as $\{H_1, H_2, H_4\}$ or $\{H_2, H_3, H_4\}$) is useless in the localization process. Indeed, the whole information on the location of the source, here the time differences τ_{ji}, can be found by measuring the 4 delays τ_{1i} when using any pair of 3-hydrophone arrays. The source location is to be found equal when considering any pair of 3-hydrophone arrays (Figure 7). The overall error on the estimated location cannot then be reduced by repeating the localization process on different sets of virtual hydrophones. Nevertheless, the detection of more surface/bottom echoes of sperm whale clicks could help.

4 RESULTS

20 clicks from a single sperm whale have been selected in a 5-minute recording. A single cable 2-hydrophone array has been used during the recording campaign in the Strait of Gibraltar, towed from the CIRCé ship Elsa. The localization method has been tested twice on two different hydrophones as to confirm the results. The delays on sperm whale click echoes have then been measured on two hydrophones, these estimated depths are plotted (Figure 3). Both hydrophones are attached to the same cable, and a common movement of the array of 1 m/min in depth is detected. A second sperm whale was clicking in the same recording, and the measurement of the delays on three of its clicks confirms the results of the depths of the hydrophones found by using the clicks of the first sperm whale.

The measurement of the delays on the 20 selected clicks on the two hydrophones enables a calculation of the depth of the source on 40 points (Figure 4). The tracked sperm whale moves downwards at a speed close to 0.2 m/s.

The results, the depth of the source and the depths of the hydrophones, are interpolated. Each calculated point is included in a band in which width represents the uncertainty on the result as calculated using the following method.

σ_τ	σ_{z_b}	σ_{c_0}	σ_{z_h}	σ_{z_s}	σ_{x_s}
0.1 ms	10 m	2 m/s	0.9 m	9.8 m	125 m

Table 1 - **Uncertainties on the measurements and the results**. The depth of the sea bottom is $z_b = -870 \pm 2\sigma_{z_b} = -870 \pm 20$ m with a 95 % probability.

5 UNCERTAINTY ESTIMATION

5.1 Theory

Let $\widehat{x}_1, ..., \widehat{x}_n$ be the measurements of $x_1, ..., x_n$ with the accuracies $\sigma_1, ..., \sigma_n$. Let $\widehat{y} = f(\widehat{x}_1, ..., \widehat{x}_n)$ be the estimated value of $y = f(x_1, ..., x_n)$ with the uncertainty σ_y. The uncertainty σ_y can be estimated from a linear uncertainty estimation method by writing :

$$\sigma_y^2 = \sum_{i=1}^{n} \left[\frac{\partial f}{\partial x_i}(\widehat{x}_1, ..., \widehat{x}_n)\right]^2 \sigma_i^2 \qquad (3)$$

σ_i is the uncertainty on the measurement $\widehat{x}_i$ of x_i, $|\partial f/\partial x_i(\widehat{x}_1, ..., \widehat{x}_n)|\sigma_i$ is the uncertainty on $\widehat{y}$ due to the uncertainty on the measurement $\widehat{x}_i$, and σ_y as given by (3) is then the quadratic mean of the uncertainties on $\widehat{y}$ due to the uncertainties on each of the measurements $\widehat{x}_i$. But errors on $\widehat{y}$ due to errors on some parameters x_i may be compensated by errors due to other parameters x_j. The linear error estimation given by (3) does not take into account such compensations. Random variables naturally do so, and may be useful to estimate the uncertainty in the result of a function of several parameters.

The uncertainties on the measurements of the parameters x_i can be modelized using random variables. Let $X_i \sim \mathrm{N}(\widehat{x}_i, \sigma_i^2)$ be n Gaussian distributed random variables distributed around the means (the measurements) $\widehat{x}_i$ with the standard deviations (the accuracies) σ_i. σ_i, representing the uncertainty on $\widehat{x}_i$, is chosen such that the real value x_i would be a sample of the random variable X_i. Then, given properties on Gaussian distributed random variables, x_i has a 68 % probability to belong to $[\widehat{x}_i - \sigma_i, \widehat{x}_i + \sigma_i]$, a 95 % probability to belong to $[\widehat{x}_i - 2\sigma_i, \widehat{x}_i + 2\sigma_i]$, and a 99 % probability to belong to $[\widehat{x}_i - 3\sigma_i, \widehat{x}_i + 3\sigma_i]$.

The uncertainty on the estimated value $\widehat{y}$ is also modelized using random variables, here $Y = f(X_1, \ldots, X_n)$. The mean, the standard deviation, and the distribution of Y can be estimated using a large number m of samples $y^{(k)}$ of Y. Each of these samples is calculated from samples $x_i^{(k)}$ of the random variables X_i, by writing

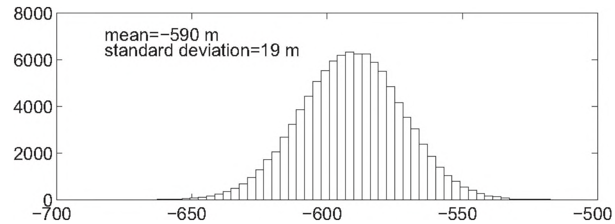

Figure 5 - **Histogram of the depth of the source**. The random variable representing the depth of the source is almost Gaussian, with a mean $z_s = -590$ m and a standard deviation σ_{z_s} =19 m. The tracked sperm whale is then at a depth $z_s = -590 \pm 38$ m with a 95 % probability.

$y^{(k)} = f(x_1^{(k)}, \ldots, x_n^{(k)})$. The mean y_m and the standard deviation σ_y of Y can then be estimated by :

$$y_m \simeq \frac{1}{N}\sum_{k=1}^{N} y^{(k)} \qquad \sigma_y^2 \simeq \frac{1}{N}\sum_{k=1}^{N}[y^{(k)} - y_m]^2 \qquad (4)$$

and its distribution is estimated by plotting a histogram of the samples $y^{(k)}$ (Figure 5).

Let σ_τ, σ_{z_b}, and σ_{c_0} be the uncertainties on the parameters τ_{1i}, z_b and c_0 (Table 1). The resulting uncertainty on $z_s = f(\tau_{1i}, z_b, c_0)$ can then be found from (4) by generating m samples $z_s^{(k)} = f(\tau_{1i}^{(k)}, z_b^{(k)}, c_0^{(k)})$ ($m = 100000$ has been drawn to plot the histogram of Figure 5). An approached value of the uncertainty σ_{z_s} found from (4) can be calculated analytically, by using some properties of Gaussian distributed random variables, as described next.

5.2 Analytic expression

Let $X_1 \sim \mathrm{N}(\widehat{x}_1, \sigma_1^2)$ and $X_2 \sim \mathrm{N}(\widehat{x}_2, \sigma_2^2)$ be two independant Gaussian distributed random variables. Then $X_1 + X_2 \sim \mathrm{N}(\widehat{x}_1 + \widehat{x}_2, \sigma_1^2 + \sigma_2^2)$ is also a Gaussian distributed random variablof mean $\widehat{x}_1 + \widehat{x}_2$ and of standard deviation $\sqrt{\sigma_1^2 + \sigma_2^2}$. σ_1 being the uncertainty on the measurement $\widehat{x}_1$, and σ_2 being the uncertainty on the measurement $\widehat{x}_2$, the uncertainty on $\widehat{x}_1 + \widehat{x}_2$ is then the quadratic mean of two uncertainties, one due to the uncertainty on $\widehat{x}_1$ (σ_1), and a second one due to the uncertainty on $\widehat{x}_2$ (σ_2). Similar evaluations of uncertainties can be calculated for different basic operations of random variables (product, ratio, square, square root). The uncertainty σ_y on $\widehat{y} = f(\widehat{x}_1, ..., \widehat{x}_n)$ can then be estimated by decomposing f in basic operations and by calculating σ_y step by step.

Analytic expressions of the uncertainties on the depth z_s of the tracked sperm whale and the depth z_h of the hy-

propagation model	τ_{12}	τ_{13}	τ_{14}
rectilinear propagation constant speed of sound	7.8 ms	100.6 ms	114.9 ms
rectilinear propagation non constant speed of sound	7.8 ms	97.0 ms	111.3 ms
non rectilinear propagation non constant speed of sound	7.7 ms	96.9 ms	111.3 ms

Table 2 - **Propagation models**. The propagation of sperm whale clicks is simulated in a configuration of the source and the receiver close to what is found from the constant speed of sound localization method. Delays on bottom echoes can here be overestimated by 3 ms when not taking into account variations in speed of sound.

drophone can then be calculated from the expressions of z_s and z_h found from (1) and (2). The uncertainties σ_{z_s} and σ_{z_h} are found as quadratic means of the uncertainties driven by the uncertainties on each of the parameters τ_{1i}, z_b, and c_0. The uncertainty in the depth of the source is found close to :

$$\sigma_{z_s}^2 \simeq z_s^2 \left[\frac{(c_0^4/16)[2\tau_{13}^2 + \tau_{23}^2]\sigma_\tau^2 + z_b^2\sigma_{z_h}^2 + (4z_b^2 + z_h^2)\sigma_{z_h}^2}{[z_b(z_b - z_h) - (c_0^2/4)\tau_{13}\tau_{23}]^2} \right.$$
$$+ \frac{(c_0^2/4)\tau_{13}^2\tau_{23}^2\sigma_{c_0}^2}{[z_b(z_b - z_h) - (c_0^2/4)\tau_{13}\tau_{23}]^2}$$
$$\left. + \frac{z_h^2[2\tau_{12}^2 + \tau_{23}^2]\sigma_\tau^2 + \tau_{12}^4\sigma_{z_b}^2 + \tau_{12}^2\tau_{23}^2\sigma_{z_h}^2}{\tau_{12}^2[\tau_{12}z_b + \tau_{23}z_h]^2} \right] \tag{5}$$

The result is nevertheless an approximation of what could be found from (3) or (4), as making strong hypotheses on the gaussianity and the independance of the parameters step after step in the calculation process.

The uncertainties on the depth of the hydrophone, the depth of the source and the range of the source are found close to each other when estimated by sampling (Table 1) or analytically ($\sigma_{z_h} = 0.8$ m, $\sigma_{z_s} = 15.5$ m and $\sigma_{x_s} = 145$ m), and close to the results (Figure 3 and Figure 4).

6 VARYING SPEED OF SOUND LOCALIZATION

6.1 Variations in the speed of sound

The previous localization and uncertainty estimation techniques assumed the speed of sound to be constant in the sea water. The value of the speed of sound in sea water depends on local conditions of temperature, salinity, pressure and current. These parameters vary, mostly with depth. The resulting variations of the speed of sound in the Strait of Gibraltar are plotted in Figure 6. The importance of the speed of sound variations on the accuracy of the previous localization method is now studied.

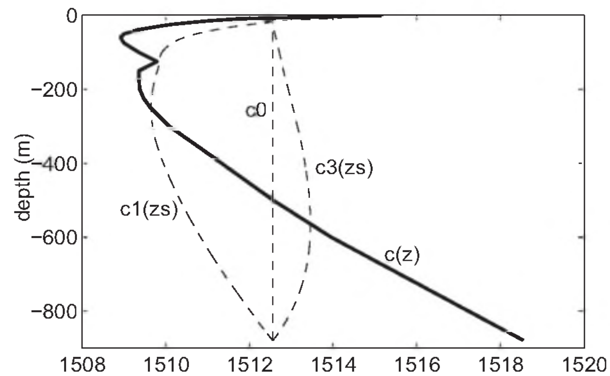

Figure 6 - **Speed of sound.** The speed of sound $c(z)$ in sea water varies mostly with depth. The mean speed of sound along the whole water column is $c_0 = 1512.6$ m/s. The mean speed of sound along the ascending rays (paths (1) and (2)) is close to $\tilde{c_1}(z_s)$. The mean speed of sound along descending rays (paths (3) and (4)) is close to $\tilde{c_3}(z_s)$. Sperm whale clicks reflected by the sea bottom will then propagate faster than sperm whale clicks reflected by the sea surface (as $\tilde{c_3}(z_s) \geq \tilde{c_1}(z_s)$).

The propagation of sound in sea water tends to be more complex with such variations in the speed of sound. Rays are curved due to refraction effects, and the speed of sound varies along the rays. The propagation times of signals from a source to a receiver, depending approximately on the length of the rays and of the mean speed of sound along the rays (as the speed of sound values are much greater than its variations), may then be wrongly estimated when supposing a constant speed of sound.

Three propagation models are compared, as to estimate the effects of the speed of sound variations (curvature of the rays, and different mean speeds of sound along the rays for different rays) on the misevaluation of the delays on received sperm whale click signals (Table 2). The curvature of the rays introduces negligible errors on the measurement of delays (the deviations of the rays from the rectilinear paths are inferior to 8 m as found by raytracing). The variations in the mean speed of sound with rays ($\tilde{c_1}(z_s) \neq \tilde{c_3}(z_s)$, Figure 6) however do induce important errors on the measurement of delays. The previous localization technique assuming a constant speed of sound may then be quite inaccurate, and is then enhanced by taking into account the variations in speed of sound.

6.2 Localization

The propagation time of a signal from a source to a receiver along a given ray is then given by the length of the ray, and by the mean speed of sound along the ray. The rays of the signals transmitted *via* the direct path and the rays of the signals reflected once by the sea surface are close to each other (the hydrophone is close to the surface). And so is the mean speed of sound along these two rays. And so it is for the rays of the two signals reflected by the sea bottom. Then, by labelling $\widetilde{c_1}$, $\widetilde{c_2}$, $\widetilde{c_3}$, and $\widetilde{c_4}$ the mean speeds of sound along the rays (1), (2), (3), and (4) (Figure 2), one can find $\widetilde{c_2}(z_s, z_h) \simeq \widetilde{c_1}(z_s, z_h) \simeq \widetilde{c_1}(z_s)$ and $\widetilde{c_4}(z_s, z_h) \simeq \widetilde{c_3}(z_s, z_h) \simeq \widetilde{c_3}(z_s)$.

The localization method is then close to the one used in the previous constant speed of sound method. Using different triplets of hydrophones, 4 polynomial equations on the depth z_s of the source and the depth z_h of the receiver are found, such as, considering $\{H_1, H_2, H_3\}$:

$$\begin{aligned}&\left[4z_b(z_b - z_s) - \frac{\widetilde{c_3}(z_s)^2 - \widetilde{c_1}(z_s)^2}{4}\tau_{12}^2 - \widetilde{c_3}(z_s)^2\tau_{13}\tau_{23}\right] + \\ &\left[-4(z_b - z_s) - 2\frac{\widetilde{c_3}(z_s)^2 - \widetilde{c_1}(z_s)^2}{\widetilde{c_1}(z_s)^2}z_s - 4\frac{\widetilde{c_3}(z_s)^2\tau_{13}}{\widetilde{c_3}(z_s)^2\tau_{12}}z_s\right]z_h + \\ &\left[-4\frac{\widetilde{c_3}(z_s)^2 - \widetilde{c_1}(z_s)^2}{[\tau_{12}\widetilde{c_1}(z_s)^2]^2}z_s^2\right]z_h^2 = 0\end{aligned} \tag{6}$$

The solution sets of these equations are plotted (Figure 7), and the intersection points are found numerically. The results are consequently close to those found by the constant speed of sound localization method (the mean deviation of the results on z_s found by this method and the constant speed of sound one, as plotted at Figure 4, is close to 1.9 m).

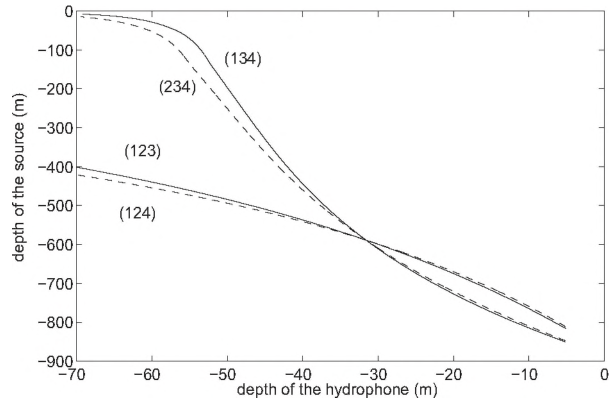

Figure 7 - **Varying speed of sound localization**. The depth z_s of the source and the depth z_h of the receiver are found from the intersection of the 4 solution sets corresponding to the 4 different triplets of hydrophones of the virtual 4-hydrophone array.

7 CONCLUSIONS

Sperm whales can be localized in range and in depth using only one hydrophone at an unknown depth. The feasibility of the method has been asserted by using a second hydrophone, by detecting clicks emitted from a second sperm whale, and by estimating the accuracy on the estimated locations using random variables. The uncertainty estimation technique developed here may be used in other localization systems using more than one hydrophone.

The depth of the sea bottom and the value of the mean speed of sound are required in the present localization method. The only requirements are the surface coordinates, for instance given by a GPS receiver, of the place of the recordings. The resulting values of the depth of the sea bottom and the speed of sound would then be given by bathymetric charts and oceanic models. The MERCATOR project enables estimations of the variations of speed of sound in the whole Mediterranean Sea. The use of past or future recordings, made with a GPS receiver and a single hydrophone, could then be used to estimate the depths of diving sperm whales.

Statistics on the diving depths of sperm whales could be made in a given area by using recordings stored in databases. The diving behaviour of sperm whales could then be better understood, in high traffic areas like the Strait of Gibraltar or the Ligurian Sea, or why not in the whole Mediterranean Sea.

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