SOUND RADIATION FROM POROELASTIC MATERIALS

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1. INTRODUCTION

Numerical approaches based on finite element discretizations of Biot's poroelasticity equations provide efficient tools to solve problems where the porous material is coupled to elastic plates and finite extent acoustic cavities. Sometimes, it may be relevant to evaluate the radiation of a poroelastic material inside an infinite fluid medium. Examples include (i) the evaluation of the diffuse field sound absorption coefficient of a porous material and/or the sound transmission loss of an elastic plate with an attached porous sheet, (ii) the assessment of the acoustic radiation damping of a porous material coupled to a vibrating structure. The latter is particularly important for the correct experimental characterization of the intrinsic damping of the material's frame. To date, the acoustic radiation of a porous medium into an unbounded fluid medium is usually neglected. The classical approach for modeling free field radiation of porous materials assumes the interstitial pressure at the radiation surface to be zero. This paper presents a numerical formulation for evaluating the sound radiation of baffled poroelastic media including fluid loading effects. The problem is solved using a mixed FEM-BEM approach where the fluid loading is accounted for using an admittance matrix solid phase-interstitial pressure coupling terms. Numerical results will be presented during the conference in order to illustrate the technique.

2. THEORY

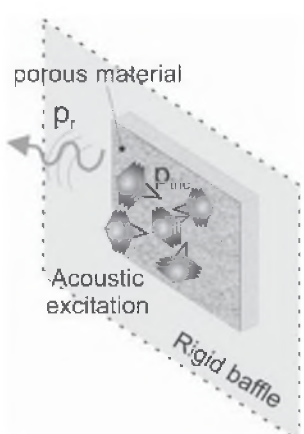

Fig.1: Configuration of the problem

Consider a rectangular porous material sample inserted into a rigid planar baffle excited acoustically. The porous material is coupled to a semi-infinite fluid on one of its face (excitation side) and has specific boundary conditions on the other faces. The modified weak integral form associated to the porous material is been given by [1]:

$$\begin{aligned}\int_{\Omega_p}\left[\underline{\underline{\tilde{\sigma}}}^s(\underline{u}):\underline{\underline{\varepsilon}}^s(\delta\underline{u})-\omega^2\tilde{\rho}\underline{u}.\delta\underline{u}\right]d\Omega+\int_{\Omega_p}\left[\frac{\phi^2}{\omega^2\tilde{\rho}_{22}}\underline{\nabla}p.\underline{\nabla}\delta p-\frac{\phi^2}{\tilde{R}}p\delta p\right]d\Omega\\-\int_{\Omega_p}\frac{\phi^2\rho_0}{\tilde{\rho}_{22}}\delta(\underline{\nabla}p.\underline{u})d\Omega-\int_{\Omega_p}\phi\left(1+\frac{\tilde{Q}}{\tilde{R}}\right)\delta(p\underline{\nabla}.\underline{u})d\Omega\\-\int_{\partial\Omega_p}\phi[\underline{U}.\underline{n}-\underline{u}.\underline{n}]\delta p d\Gamma-\int_{\partial\Omega_p}\left[\underline{\underline{\sigma}}^t.\underline{n}\right].\delta\underline{u}d\Gamma=0\ \ \forall\ (\delta\underline{u},\ \delta p)\end{aligned}\qquad(1)$$

Ω_p and $\partial\Omega_p$ refer to the porous-elastic domain and its bounding surface. $\underline{u}$ and p are the solid phase displacement vector and the interstitial pressure in the porous-elastic medium, respectively. $\underline{U}$ is the fluid macroscopic displacement vector. $\delta\underline{u}$ and δp refer to their admissible variation, respectively. $\underline{n}$ denotes the unit normal vector external to the bounding surface $\partial\Omega_p$. ϕ stands for the porosity, $\tilde{\rho}_{22}$ is the modified Biot's density of the fluid phase accounting for viscous dissipation, $\tilde{\rho}$ is a modified density given by $\tilde{\rho}=\tilde{\rho}_{11}-\frac{\tilde{\rho}_{12}}{\tilde{\rho}_{22}}$ where $\tilde{\rho}_{11}$ is the modified Biot's density of the solid phase accounting for viscous dissipation. $\tilde{\rho}_{12}$ is the modified Biot's density which accounts for the interaction between the inertia forces of the solid and fluid phase together with viscous dissipation. $\underline{\underline{\tilde{\sigma}}}^s$ and $\underline{\underline{\varepsilon}}^s$ are the in-vacuo stress and strain tensors of the porous material. $\underline{\underline{\tilde{\sigma}}}^t$ is the total stress tensor of the material given by: $\underline{\underline{\tilde{\sigma}}}^s=\underline{\underline{\sigma}}^t+\phi\left[1+\frac{\tilde{Q}}{\tilde{R}}\right]p\underline{\underline{1}}$. Note that $\underline{\underline{\tilde{\sigma}}}^s$ accounts for structural damping in the skeleton through a complex Young's modulus $E(1+j\eta_s)$. $\tilde{Q}$ is an elastic coupling coefficient between the two phases, $\tilde{R}$ may be interpreted as the bulk modulus of the air occupying a fraction ϕ of the unit volume aggregate.

In this formulation, the porous media couples to the semi-infinite fluid medium through the following boundary terms:

$$-\int_{\partial\Omega_p}\left[\underline{\underline{\sigma}}^t.\underline{n}\right]\delta\underline{u}d\Gamma-\int_{\partial\Omega_p}\phi\left(U_n-u_n\right)\delta pd\Gamma \quad (2)$$

Since $\underline{\underline{\sigma}}^t.\underline{n}=-p\underline{n}$, at the excited surface, (2) becomes:

$$\int_{\partial\Omega_p}\delta\left(pu_n\right)d\Gamma-\int_{\partial\Omega_p}\left[\phi\left(U_n-u_n\right)+u_n\right]\delta pd\Gamma \quad (3)$$

In the semi-infinite domain, the acoustic pressure p_a is the sum of the blocked pressure p_b and the radiated pressure p_r. Applying the continuity of the normal displacement at the surface, (3) becomes:

$$\int_{\partial\Omega_p}\delta\left(pu_n\right)d\Gamma-\frac{1}{\rho_0\omega^2}\int_{\partial\Omega_p}\frac{\partial p_a}{\partial n}\delta pd\Gamma \quad (4)$$

Since $\partial p_a/\partial n=\partial p_r/\partial n$, the associated discrete form to the second term of (4) is:

$$\frac{-1}{\rho_0\omega^2}\int_{\partial\Omega_p}\frac{\partial p_a}{\partial n}\delta pd\Gamma=\frac{-1}{\rho_0\omega^2}\langle\delta p\rangle[C]\left\{\frac{\partial p_r}{\partial n}\right\} \quad (5)$$

where [C] is a classical coupling matrix given by $\int_{\partial\Omega_p}\langle N(M)\rangle\{N(M)\}d\Gamma(M)$ and $\{N(M)\}$ denotes the vector of the shape functions. The porous material being inserted into a rigid baffle, the acoustic pressure is related to the normal velocity using Rayleigh's integral:

$$p_r(x,y,z)=-\int_{\partial\Omega_p}\frac{\partial p_r(x,y,0)}{\partial n}G(x,y,0,x',y',0)d\Gamma \quad (6)$$

where $G(x,y,0,x',y',0)=\frac{e^{-jk_0R}}{2\pi R}$ is the baffled Green's function, $k_0=\omega/c_0$, is the acoustic wave number in the fluid, c_0, the associated speed of sound and R is defined by $R=\sqrt{(x-x')^2+(y-y')^2}$.

An associated integral form to (6) is given by:

$$\int_{\partial\Omega_p}p_r(x,y,z)\delta pd\Gamma=-\int_{\partial\Omega_p}\int_{\partial\Omega_p}\frac{\partial p_r(x,y,0)}{\partial n}G(x,y,0,x',y',0)\delta pd\Gamma \quad (7)$$

The associated discrete form is:

$$\langle\delta p\rangle[C]\langle p_r\rangle=-\langle\delta p\rangle[Z]\left\{\frac{\partial p_r}{\partial n}\right\} \quad (8)$$

with

$$[Z]=\int_{\partial\Omega_p}\int_{\partial\Omega_p}\langle N(M)\rangle G(M,M')\{N(M')\}d\Gamma(M)d\Gamma(M')$$

Since $\langle\delta p\rangle$ is arbitrary, one gets:

$$\left\{\frac{\partial p_r}{\partial n}\right\}=-[Z]^{-1}[C]\langle p_r\rangle \quad (9)$$

Substituting (9) into (5), and recalling that on the interface $p=p_a=p_r+p_b$, the discrete form of (3) reads finally:

$$\langle\delta u_n\rangle[C]\{p\}+\langle\delta p\rangle[C]^T\{u_n\}-\frac{1}{j\omega}\langle\delta p\rangle[A]\{p-p_b\} \quad (10)$$

where $[A]=\frac{1}{j\omega\rho_0}[C][Z]^{-1}[C]$ is an admittance matrix. The radiation of the porous medium into the semi infinite fluid amounts to an added admittance term onto the interface interstitial pressure degrees of freedom and to additional interface coupling terms between the solid phase and the interstitial pressure (first terms in (10)). The last term involving P_b is the excitation term.

Using classical notations [1], the discretized form of (1) combined with (10) leads to the following linear system:

$$\begin{bmatrix}-\omega^2[\tilde{M}]+[K] & -[\tilde{C}]+[C]\\ -[\tilde{C}]^T+[C]^T & \frac{[\tilde{H}]}{\omega^2}-\frac{[\tilde{A}]}{j\omega}-[\tilde{Q}]\end{bmatrix}\begin{Bmatrix}u\\p\end{Bmatrix}=\begin{Bmatrix}0\\F_f\end{Bmatrix} \quad (11)$$

with $\{F_f\}=\frac{1}{j\omega}[A]\{p_b\}$.

This system is first solved in terms of the porous solid phase nodal displacements and interstitial nodal pressures. Next, the vibroacoustic indicators of interest can be calculated.

3. CONCLUSION

This paper presented an approach to predict the sound radiation of baffled poroelastic media including fluid loading effects. The problem has been solved using a mixed FEM-BEM approach where the fluid loading is accounted for using an admittance matrix and solid phase-interstitial pressure coupling terms. The method has been considered in the case of an acoustic excitation but the approach is general and can be used as soon as the porous material is inserted in a rigid baffle and radiates into a semi infinite fluid. Numerical examples will be presented during the oral presentation in order to illustrate the technique.

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