ON THE STOCHASTIC PROPERTIES OF THE NEURAL ENCODING MECHANISM OF SOUND INTENSITY

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1. INTRODUCTION

One of the simplest ways to model the activity in a neural spike train is to use a Poisson process. However, the conventional homogeneous Poisson process (HPP) is not compatible with the results collected from past studies. For example, the pulse-number distribution (PND) extracted from an HPP will follow a Poisson distribution with a mean-to-variance ratio of one. On the other hand, several studies have indicated that the ratio is not unity but is approximately equal to 2 across the dynamic range (e.g. Teich and Khanna, 1985). Additionally many features in the behaviour of real sensory neurons such as rate adaptation, rate-intensity dependence and a dead time in spike activity require that modifications be made to this model.

Based on these concerns, the HPP was modified and extended to give a more realistic stochastic model that expressed quantitatively the properties of auditory neurons. The predictions of the model with respect to the mean-to-variance ratio will be taken as an indication of whether the new model outperforms the conventional HPP-based model.

2. METHOD

In an HPP, the inter-event intervals $\mathbf{t}_{1,2,...}$ which specify (in our case) the interval been spikes are governed by independent exponential random variables with a probability density function $f(\mathbf{t}_n = x) = \boldsymbol{\lambda}\,\mathbf{T}\,e^{-\boldsymbol{\lambda}\mathbf{T}x}$ (Leon-Garcia, 1994). $\boldsymbol{\lambda}$ is a constant representing the spike count within a fixed time window **T**. The exponential distribution was used as a basis from which the new model was developed.

2.1 Firing Rate Modifications

We discarded the fixed spike rate $\boldsymbol{\lambda}$ and used in its place a rate function $\boldsymbol{\lambda}(\mathbf{L}, \mathbf{t})$, where **L** is sound intensity level and **t** is stimulus duration. This function was constructed based on the measurements of rate adaptation (Litvak, *et al.*, 2003) and intensity dependence (Yates, *et al.*, 2000; Smith, 1988). Please see Figure 1. With a non-constant firing rate, the process we have described is known as a non-homogeneous Poisson process (NHPP).

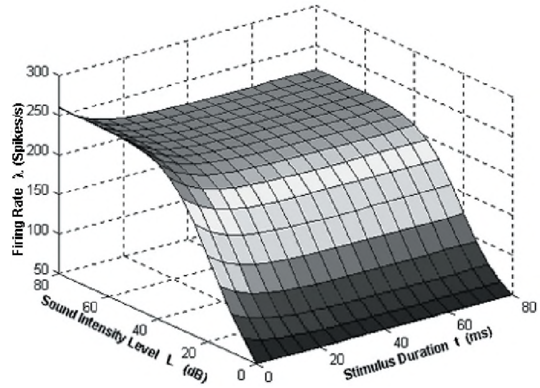

Figure 1. The idealised firing rate behaviour with respect to stimulus duration and sound intensity level for a peripheral neuron.

2.2 Dead Time Modifications

A fixed value $\boldsymbol{\tau}$ was used to denote the dead time during which the neuron cannot be activated further. In our model, the inter-spike interval was set equal to the sum of the time value generated by the firing probability function and the dead time $\boldsymbol{\tau}$. This process is known as a dead-time-modified Poisson Process (DTMPP).

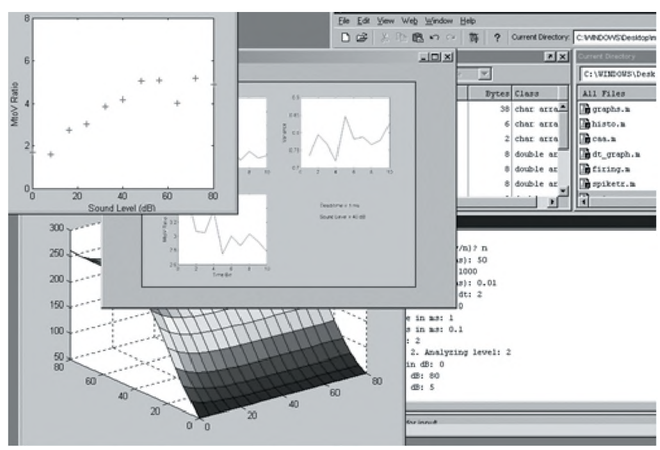

Figure 2. A screen shot of the program in MATLAB.

We implemented our stochastic model within MATLAB. To study the effects of the different components on the mean-variance ratio, a simple command program was written in MATLAB to control the different parameter values (Figure 2). A flowchart illustrating the difference between the various simulations is shown in Figure 3. Each trial spans over 50 ms and inter-spike intervals were generated with the appropriate firing probability equation. The same trial was repeated 1000 times to collect sufficient data for parameter estimation and PND generation.

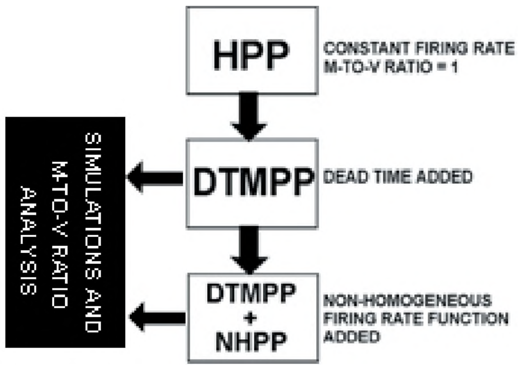

Figure 3. A diagram illustrating the process of model construction and the subsequent simulations and statistical analysis.

3. RESULTS

The results are presented in the form of plots of mean-to-variance ratio against different parameters.

3.1 Dead Time Effects under Constant Firing Rate

We considered the effect of dead time with firing rate as a parameter (Figure 4). Under a fixed firing rate, the mean-variance ratio was found to be a monotonic increasing function of dead time. Furthermore, the mean-to-variance ratio grows with increasing values of firing rate. This dependence becomes stronger as the dead time is increased.

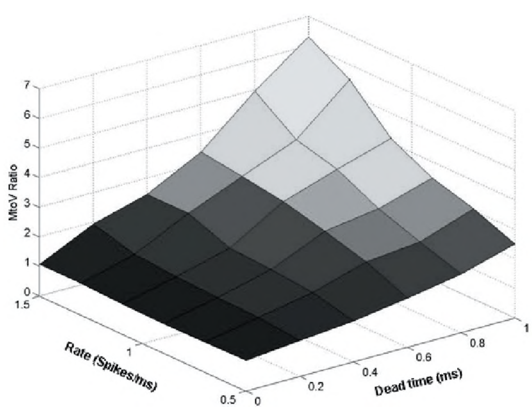

Figure 4. M-to-V ratio versus mean firing rate and dead time.

3.2 Dead Time with Time-varying Firing Rate

The full rate function was used here. Figure 5 shows that the mean-to-variance ratio behaves in a similar manner as an HPP model with dead time. The ratio remains around one without dead time, and is driven above one when a dead time is present. However, the growth is not as rapid as when plotted against the average firing rate. In fact, under a moderate dead time (0.5 ms), the ratio demonstrates little dependence on sound intensity level and appears to be around 2 in agreement with results from Teich and Khanna (1985) .

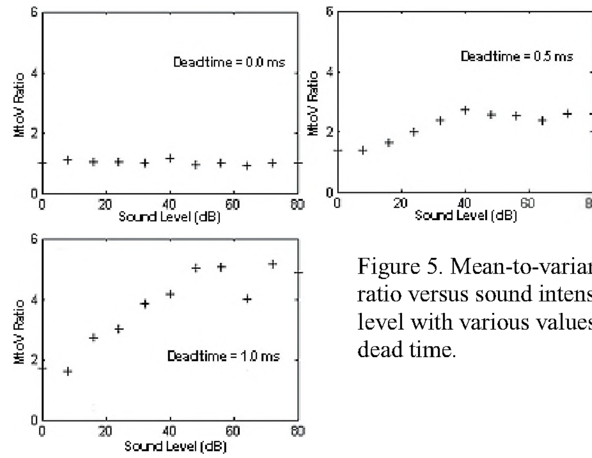

Figure 5. Mean-to-variance ratio versus sound intensity level with various values of dead time.

4. DISCUSSION

Clearly dead time is the dominant factor that drives the mean-to-variance ratio above unity. In fact, in a DTMPP the mean-variance ratio can be shown to equal approximately $(1 - \lambda \tau)^{-2}$ (Cantor and Teich, 1975). With a non-zero dead time, the M-V ratio is positively correlated with spike rate. However using a time-varying rate function, the ratio does not grow significantly when plotted against sound intensity. This is most certainly due to the fact that since the growth rate in mean activity is diminished with adaptation, the effects of sound level on the M-V ratio are minimized.

Using a moderate dead time value, we have found that the values obtained for the mean-to-variance ratio are compatible with existing experimental results. However, further investigation will be required to thoroughly evaluate this model with other intensity-coding properties of auditory neurons.

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