EXTENSION OF THE AGGREGATE BEAMFORMER TO FILTER-AND-SUM BEAMFORMING

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1. INTRODUCTION

Beamforming combines signals from an array of sensors to obtain a directional response. In basic delay and sum (D&S) beamforming, the signal from each sensor is delayed to contribute constructively to the summed output. Delays are determined by the propagation time across the array. Sources that are off-beam (not in the desired direction) are partially cancelled by incoherent summation, whereas on-beam sources (in the 'steering' direction) are enhanced by coherent summation. The directional response can be adjusted by applying weights to sensor inputs. Moreover, in a more general approach referred to as filter-and-sum (F&S) beamforming, sensor inputs can be filtered to apply a frequency dependent weight and delay (phase), allowing more general optimization.

The aggregate beamformer differs from conventional beamforming in that it converts off-beam signals to noise that can then be filtered. The array is sampled, one sensor at a time, at a relatively high sampling rate. For each sample, the sensor is selected randomly. The sampled data are time-aligned (as in conventional beamforming), low-pass filtered, and then decimated to the Nyquist frequency of the desired bandwidth. The time-aligned data faithfully reconstructs on-beam signal components but reduces off-beam components to random noise.

Aspects of the aggregate beamformer are described in articles by the author [1-4] in the context of delay-and-sum beamforming. In this article, the technique is extended to filter-and-sum beamforming (as suggested in [3]) through the introduction of a 'stochastic filter'. The stochastic filter, similar to the aggregate beamformer, reduces out-of-band signals to random noise while preserving in-band signals.

2. D&S AGGREGATE BEAMFORMING

Conventional D&S beamforming for an array of M sensors, each with a sampled input signal $x_m(n)$, has output

$$B_C(n) = \sum_{m=1}^{M} w_m x_m(n - \tau'_m) , \tag{1}$$

where w_m are the 'shading' weights and τ'_m are the beamformer sample delays. The corresponding formulation of the aggregate beamformer output is

$$B_A(n) = \sum h^d(m) x_{s(m')}(m' - \tau_{\sigma(m')}) , \tag{2}$$

where we have defined the implicit summation index $m' = (nK_{os}-m)$ for brevity. For the aggregate beamformer, $h^d(m)$ are decimation filter coefficients, $K_{os} \geq 1$is a decimation factor, $t_m = t'_m/K_{os}$ are sample delays and $\sigma(n)$ is a random sequence of sensor indices that are independently and identically distributed as

$$\Pr\{\sigma(n) = m\} = w_m . \tag{3}$$

($\Pr\{\cdot\}$ indicates probability.)

Array weights w_m are applied by adjusting the sampling probabilities; no scaling of data is necessary. The residual noise level is controlled by adjusting the over-sampling factor K_{os} and can also be reduced by noise shaping [4].

3. STOCHASTIC FILTERING

The output $y_c(n)$ of a conventional FIR filter with impulse response $\{h(m)\}$ $(0 \leq m \leq M)$ is

$$y_c(n) = \sum_m h(m) x(n-m) . \tag{4}$$

We define the output $y_s(n)$ of the corresponding stochastic filter as

$$y_s(n) = \varepsilon_{\rho(n)} x(n - \rho(n)) , \tag{5}$$

where ρ is a random sequence of sample indices that are independently and identically distributed with

$$\Pr\{\rho(n) = m\} = |h(m)| \quad \text{and} \quad \varepsilon_m = \frac{h(m)}{|h(m)|} . \tag{6}$$

It is assumed that the FIR coefficients satisfy $\sum |h(m)| = 1$. The conventional and stochastic FIR filters are equivalent in the mean (both in magnitude and phase);

$$\langle y_s(n) \rangle = y(n) . \tag{7}$$

where $\langle \cdot \rangle$ indicates expected value. Both the conventional and stochastic filters are linear but the stochastic filter is only time invariant in the mean. We define the residual noise of the stochastic filter as

$$\eta(n) = y_s(n) - y_c(n). \quad (8)$$

It can be shown that η has zero mean and is spectrally white irregardless of the input signal. If the input signal is stationary then the total output power is equal to the input signal power. The SNR at the output depends upon how much of the input signal is in-band but is bounded above by

$$\frac{\sum h_m^2}{1 - \sum h_m^2}. \quad (9)$$

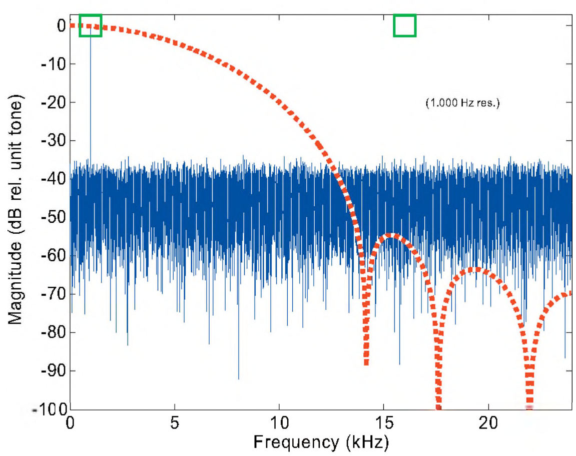

Fig. 1 *Stochastic filter output spectrum. The input tones (large squares) are 1 kHz and 16 kHz. The dashed curve is the filter magnitude response.*

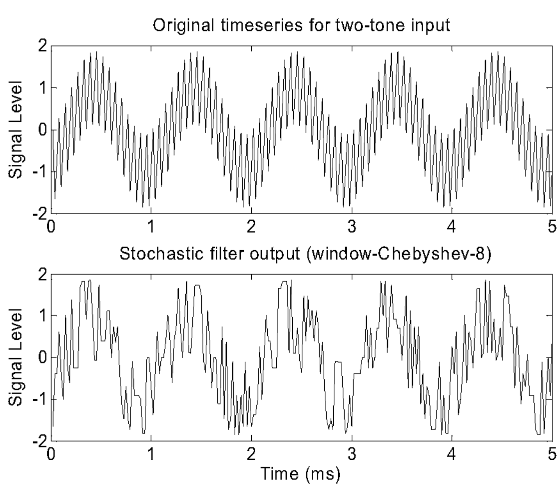

Fig. 2 *Stochastic filter input and output time series.*

Figure 1 shows the output magnitude spectrum of a stochastic filter for an input consisting of two equal-amplitude tones. The filter is an 8th order Chebyshev window filter, which has no negative coefficients. The dotted curve is the frequency response of the conventional FIR filter. The large squares indicate the components of the input. The lower frequency component is passed by the stochastic filter while the higher frequency component is converted to white noise.

The input and output time series of the example stochastic filter are shown in Fig. 2.

4. F&S Aggregate Beamformer

The filter-and-sum aggregate beamformer output B_{AFS} is derived from the stochastic filter by combining the random indices defined in Eqs. (3) and (6),

$$B_{\mathrm{AFS}}(n) = \sum_m h^{\mathrm{d}}(m) x_{\sigma(m')}\left(m' - \tau_{\sigma(m')} - \rho(m')\right). \quad (10)$$

We have again used the implicit index $m' = (nK_{\mathrm{os}} - m)$ in the summation. Recall that $h^{\mathrm{d}}(m)$ are decimation filter coefficients and that the stochastic filter coefficients are incorporated through $\rho(m')$.

Unlike conventional beamforming, extending D&S aggregate beamforming to F&S does not incur additional computation. The residual noise level, however, may be significantly increased.

5. CONCLUSION

The delay and sum aggregate beamformer can be extended to filter and sum beamforming. The conventional and the aggregate F&S beamformers are equivalent in the mean. The residual noise in the F&S beamformer may be significantly greater than the D&S beamformer.

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