BEAMFORMING A BENT ARRAY

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1. INTRODUCTION

The application of interest is the processing of a towed array that continuously changes shape as the tow ship maneuvers. It is assumed that the deviation of the array shape from a linear geometry can be so large that it is desirable to compensate for the array shape within the beamformer. In such a system, the signal-processing algorithms and display layouts must be designed to handle the changing array shape without human intervention. In this paper, we treat such issues as defining the number of beams and their steering angles, as well as the reference line from which these angles are measured.

2. BEAMFORMING [1]

It is assumed that the towed array is confined to the horizontal plane. The positions of the hydrophones will be denoted by (x_n, y_n) for $n = 0, \ldots, N-1$. Figure 1 illustrates the geometry in plan view. Here the x-axis is shown aligned with the axis of the tow ship, but the exact positioning of the coordinate system is not important for the following theory.

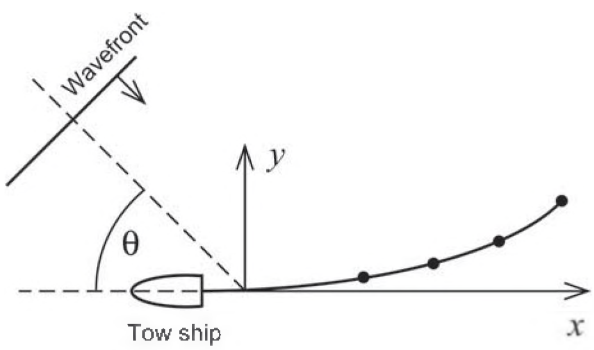

Fig. 1. Plan view of the towed array and coordinate system.

We consider the beam-pattern of a narrowband beamformer processing a signal of spatial dependence $\exp[-i\omega\tau_n(\theta)]$, where ω is the frequency and $\tau_n(\theta)$ the propagation delay between the coordinate origin and the n-th hydrophone for an arrival angle θ. Angles are measured with respect to the negative x-axis, and range between $\pm 180^{\circ}$. For this plane-wave signal, $\tau_n(\theta) = (x_n \cos\theta - y_n \sin\theta)/c$, where c is the speed of sound. The beam-pattern for the m-th beam ($m = 0, \ldots, M-1$) is given by $|b(\theta;\theta_m)|^2$, where

$$b(\theta;\theta_m) = \frac{1}{W}\sum_{n=0}^{N-1} w_n \exp[i\omega\{\tau_n(\theta_m) - \tau_n(\theta)\}] \qquad (1)$$

and $\tau_n(\theta_m)$ is the delay inserted into the beamformer to steer the m-th beam at angle θ_m. Also, the w_n are hydrophone weights, with sum W.

For an operational sonar system, there is strong motivation to minimize the number of beams while avoiding gaps in detection coverage, such as occur if the beams are spaced too far apart ("scalloping" loss, or "picket fence" effect). Most of the time the tow ship is on a steady course, for which the array is approximately linear, and hence the number of beams and their steering angles would be optimized for the linear geometry. Our purpose is to see how well the coverage is maintained during a tow-ship maneuver, when the array shape is distorted. As will be seen, in order to reduce scalloping loss it is important to choose an appropriate reference axis for the steering angles.

3. REFERENCE AXIS

Given the hydrophone positions, an obvious approach to defining a reference baseline is to apply linear regression. However, the orientation of the line in physical space should not depend on the coordinate system in which it is computed. The standard method of linear regression [2] does not satisfy this property, because deviations from the line are measured in the y-direction. Hence a rotation of the coordinate system, changing the y-direction, also changes the regression solution. An approach that is independent of the coordinate system is to find the line that minimizes the sum of squares of the *perpendicular* deviations from the line. This method of regression, which we call "p-directed", is quite old [3]; see also [4–5].

Although the hydrophone positions are assumed to be known at any instant of time, and are not random in any sense, it will nevertheless be useful to introduce the language and notation of statistics. We define a weighted mean $\overline{x} = \frac{1}{W}\sum_n w_n x_n$, with similar definitions for $\overline{y}$, $\overline{xy}$, etc. The variance and covariance are then defined as $\mathrm{var}(x) = \overline{x^2} - (\overline{x})^2$ and $\mathrm{cov}(x,y) = \overline{xy} - \overline{x}\cdot\overline{y}$. It can be shown [3–5] that the slope of the p-directed regression line relative to the x-axis is $\tan\psi$, where the angle ψ is given by

$$\psi = \frac{1}{2}\arctan\left[\frac{2\,\mathrm{cov}(x,y)}{\mathrm{var}(x)-\mathrm{var}(y)}\right]. \qquad (2)$$

In many cases p-directed regression gives a numerical value of the slope differing little from the conventional regression slope, which is $\tan\psi = \mathrm{cov}(x,y)/\mathrm{var}(x)$. However, p-directed regression has properties convenient for theoretical analysis; for example, if the x-y coordinate system is rotated by the angle ψ in Eq. (2) to define new hydrophone positions (x'_n, y'_n), then $\mathrm{cov}(x', y') = 0$.

4. EXAMPLE

In this example, the towed array comprises 100 hydrophones spaced 2 m apart. The beam patterns shown below are for a narrowband signal at frequency 350 Hz, slightly below the array design frequency of 375 Hz (for c = 1500 m/s). The weights w_n used in the beamformer are Kaiser-Bessel weights with –30 dB sidelobe suppression.

When the array is linear, good coverage is provided by M = 99 beams spaced equally in cosine of angle over 0^o to 180^o. Mathematically, the angles relative to the array axis are $\theta_m = \arccos(1-2m/98)$ for $m = 0,\ldots,98$. In the linear case, the cross-over point of any two adjacent beams has the same numerical value, –1.8 dB (the maximum scalloping loss). Now suppose that the towed array lies on an arc of a circle of diameter 1 km. The head of the array is assumed tangent to the x-axis, as illustrated in Fig. 2. To allow for the bent array, where 360^o coverage is necessary, we steer at angles $\pm\theta_m$ relative to a given reference axis (using the same θ_m as for the linear array).

When the x-axis is used as the reference axis for the bent array, the beam coverage is that of Fig. 3(a), which shows the *envelope* of the patterns of all the beams. The maximum scalloping loss is seen to vary with angle, being smaller than necessary in some regions and larger than desired in others. Figure 3(b) shows the beam coverage when the steering angles are taken relative to the regression line, which is at an angle $\psi = 11.3^o$ to the x-axis. The coverage is now much more uniform: the maximum scalloping loss is almost the same as for the linear array over the entire circle, although it dips slightly below –2 dB near the endfire directions. (The regression line is used here to define the endfire directions of the curved array.)

5. SUMMARY

A good choice of reference axis has been described for beamforming a bent towed array when the number of beams and their steering angles are optimized for the linear geometry. In particular, the use of an appropriate reference axis can reduce the loss in detection coverage that results from scalloping.

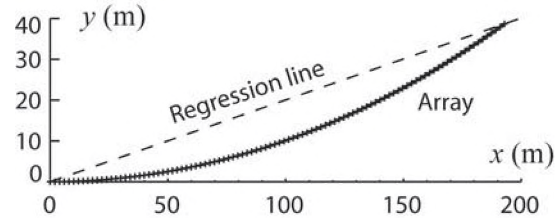

Fig. 2. The curved array and its regression line. The line has been translated so that it goes through the origin.

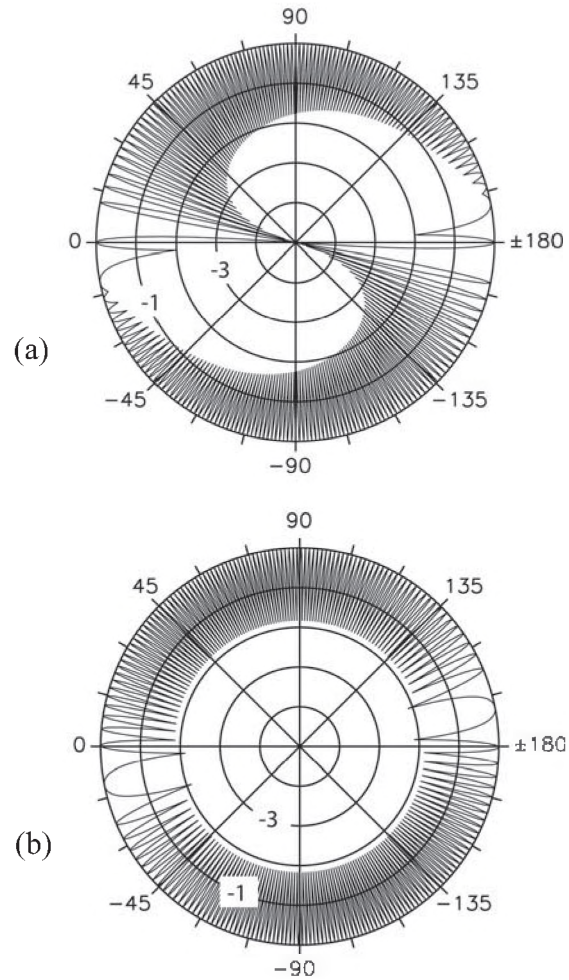

Fig. 3. Polar plots of the envelope of the beam-patterns for the bent array. Steering angles are relative to (a) the x-axis and (b) the regression axis. Beam peaks are at 0 dB and rings at –1-dB intervals.

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