DATA ERROR ESTIMATION IN MATCHED-FIELD GEOACOUSTIC INVERSION

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1. INTRODUCTION

The problem of estimating seabed geoacoustic parameters by inverting measured ocean acoustic fields has received considerable attention in recent years. Matched-field inversion (MFI) is based on searching for the set of geoacoustic model parameters **m** that minimizes an objective function quantifying the misfit between measured and modelled acoustic fields. A number of approaches have been applied to this challenging nonlinear optimization problem. In particular, adaptive simplex simulated annealing (ASSA) [1], a hybrid optimization algorithm that combines local (gradient-based) downhill simplex moves within a fast simulated annealing global search, has proved highly effective for MFI.

In a Bayesian formulation of MFI, the objective function to be minimized is derived from the likelihood function corresponding to the assumed data uncertainty distribution. The likelihood depends on parameters describing data uncertainties (e.g., standard deviations) which are nuisance parameters in terms of recovering seabed properties, but must be accounted for in a rigorous inversion. Data uncertainties include both measurement errors (e.g., due to instrumentation and ambient noise) and theory errors (due to the simplified model parameterization and approximate acoustic propagation model). Theory errors in particular are generally not well known, and tend to increase with frequency due to the effects of scattering, 3-D environmental variability, sensor location errors, etc. [2]. This paper derives several likelihood-based objective functions and examines their performance in MFI of acoustic data with unknown, frequency-dependent uncertainties.

2. THEORY

For acoustic data $\mathbf{d}_f$ measured at an N-sensor array at $f=1,F$ frequencies contaminated by independent, complex Gaussian-distributed errors with standard deviations σ_f, it can be shown that the likelihood function when source amplitude and phase are unknown is given by [2]

$$L(\mathbf{m},\boldsymbol{\sigma}) = \prod_{f=1}^{F} \frac{1}{(\pi\sigma_f^2)^N} \exp[-(1-B_f(\mathbf{m}))\,|\,\mathbf{d}_f\,|^2\,/\,\sigma_f^2\,],$$

where

$$B_f(\mathbf{m}) = |\,\mathbf{d}_f(\mathbf{m})^{\mathrm{T}}\mathbf{d}_f\,|^2\,/\,|\,\mathbf{d}_f(\mathbf{m})\,|^2|\,\mathbf{d}_f\,|^2$$

is the normalized Bartlett (linear) correlator, $\mathbf{d}_f(\mathbf{m})$ are the data predicted for model **m**, and $^{\mathrm{T}}$ denotes conjugate transpose. Maximum-likelihood parameter estimates are obtained by maximizing the likelihood over **m**. If the standard deviations σ_f are *known*, this is equivalent to minimizing the objective function

$$E_1(\mathbf{m}) = \sum_{f=1}^{F}(1-B_f(\mathbf{m}))\,|\,\mathbf{d}_f\,|^2\,/\,\sigma_f^2\,.$$

However, as mentioned above, data uncertainties are rarely well known due to theory errors. The standard approach is to assume that the uncertainty weighting factor $|\mathbf{d}_f|^2/\sigma_f^2$ is *uniform* over frequency, and minimize an objective function

$$E_2(\mathbf{m}) = \sum_{f=1}^{F}\left[1-B_f(\mathbf{m})\right].$$

However, this is often a poor assumption in practice [2]. A straightforward approach for unknown uncertainties is to *explicitly* estimate the standard deviations as part of the inversion by minimizing the objective function

$$E_3(\mathbf{m},\boldsymbol{\sigma}) = \sum_{f=1}^{F}\left[(1-B_f(\mathbf{m}))\,|\,\mathbf{d}_f\,|^2\,/\,\sigma_f^2 + N\ln\sigma_f^2\right]$$

over **m** and **σ**. The disadvantage to this approach is that it introduces F new unknown parameters σ_f, resulting in a more difficult inverse problem. An alternative approach is to maximize the likelihood over σ_f, yielding the analytic solution

$$\sigma_f = (1-B_f(\mathbf{m}))\,|\,\mathbf{d}_f\,|^2\,/\,N\,.$$

Substituting this back into the likelihood function leads (after some algebra) to an objective function

$$E_4(\mathbf{m}) = \prod_{f=1}^{F}(1-B_f(\mathbf{m})).$$

Minimizing this objective function treats the data standard deviations as *implicit* unknowns without increasing the number of parameters in the inversion.

3. RESULTS

This section considers a synthetic study of inversion performance for the various objective functions based on a shallow-water geoacoustic experiment carried out in the

Mediterranean Sea southeast of Elba Island [2]. The seabed model consists of a sediment layer over a semi-infinite basement. The unknown geoacoustic parameters are the sediment thickness, h, and sound speed, attenuation and density of the sediment and of the basement, c_1, α_1, ρ_1 and c_2, α_2, ρ_2, respectively. In addition, small corrections to the water depth, D, and source range and depth, r and z, are also included in the inversion as these geometric parameters are generally not know to sufficient accuracy. Synthetic acoustic fields were generated at 50-Hz intervals from 300-500 Hz, with Gaussian noise added so that the signal-to-noise ratio decreased uniformly from 12 to 0 dB across the band, reflecting the observed increase in theory error with frequency [2].

ASSA inversions were carried out for 200 different realizations of random noise on the data using each of the four objective functions. In all cases the inversion found a set of model parameters with an objective function value less than that for the true parameters, indicating an excellent solution. The explicit formulation (objective function E_4) required about three times more forward model calculations for convergence than the other cases due to the increased number of unknowns. The inversion results are quantified in terms of standard deviations about the true model parameters. The model-parameter standard deviations for the case where the data standard deviations are known exactly (objective function E_1) are given in Table 1. These results indicate that, relative to the parameter search bounds, the basement sound speed is the most accurately estimated parameter, followed by the sediment thickness, sediment sound speed, and basement attenuation. The other parameters have relatively large standard deviations and are poorly undetermined in the inversion

To compare inversion performance for the various objective functions, the model-parameter standard deviations obtained in all cases were divided by those obtained for objective function E_1 (known data standard deviations) to obtain normalized errors, which are shown in Fig. 1. This figure shows that objective function E_2, based on the assumption of uniform data uncertainty weighting, leads to normalized errors greater than unity for all parameters, with particularly large errors for the well-determined parameters. By contrast, objective functions E_3 and E_4 (which explicitly and implicitly include data standard deviations in the inversion, respectively) produce much smaller normalized errors which are close to unity for all parameters.

These results suggest that the acoustic data contain sufficient information content to estimate data standard deviations as well as (some) geoacoustic parameters, and that including standard deviations in the inversion (explicitly or implicitly) is preferable to the standard assumption of uniform data uncertainty weighting over frequency. Further, the implicit formulation requires no greater computational burden than the standard approach, but produced significantly better results.

Table 1. True parameter values, search bounds, and model-parameter standard deviations obtained using objective function E_1 (known data standard deviations).

Parameter	True Value & Search Bounds	Stnd Dev
h (m)	7 [0- 30]	1.5
c_1 (m/s)	1495 [1460 -1550]	15
c_2 (m/s)	1530 [1500-1600]	4.0
α_1 (dB/λ)	0.1 [0-0.5]	0.2
α_2 (dB/λ)	0.2 [0-0.5]	0.1
ρ_1 (g/cm^3)	1.4 [1.0-1.8]	0.25
ρ_2 (g/cm^3)	1.6 [1.2-2.2]	0.30
D (m)	130 [128-132]	1.5
r (km)	3.9 [3.8-4.0]	0.05
z (m)	10 [8-14]	2.0

REFERENCES

[1] S.E. Dosso, M.J. Wilmut & A.-L.S. Lapinski, 2001. An adaptive hybrid algorithm for geoacoustic inversion, *IEEE J. Oceanic Eng.*, **28**, 24-336.

[2] S.E. Dosso & P.L. Nielsen, 2002. Quantifying uncertainty in geoacoustic inversion, II. Application to broadband, shallow-water data, *J. Acoust. Soc. Am.*, **111**, 143-159.

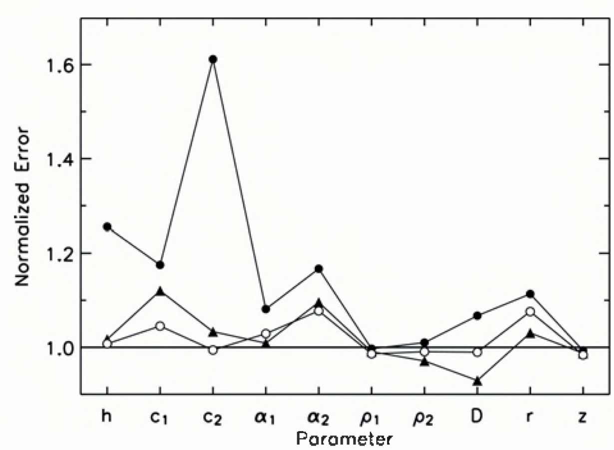

Fig. 1. Normalized parameter standard deviations for uniform, explicit, and implicit processors (filled circles, triangles, and open circles, respectively).