Assessment Of The Solution And Prediction Algorithms During The Optimization Of Fluid-Structure Interaction Dynamic Systems

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ABSTRACT

For optimization problems based on dynamic criteria the system eigenvalues must be re-computed for each iteration as the values of the design parameters are changed. From a computational point of view it would be more efficient to replace the laborious process of determining the eigenvalues by direct prediction. The suitability and advantages of this scheme are examined here. The number of operations required by the direct and the predictive solution algorithms are compared. The prediction scheme has been applied to the problem of maximizing the separation of two adjacent eigenvalues for structural and couple fluid-structure systems.

SOMMAIRE

Les problèmes d'optimisation basés sur des critères dynamiques doivent obtenir les valeurs propres de système, qui dépendent directement des valeurs des variables de conception. Pendant le processus d'optimisation la fonction objective est calculée à plusieurs reprises pour chacun nouvel ensemble de variables de conception, et alors une alternative plus économique du point de vue informatique devrait prévoir les valeurs propres pour le nouvel ensemble de variables au lieu de résoudre le problème encore. Ainsi, le but de ce travail est de déterminer la convenance et les avantages d'employer la prévision de valeurs propres, au lieu des solutions directes, dans les itérations pendant le processus d'optimisation. Puis, le nombre d'opérations entre la solution *directe* et *prédictive* du système est comparé pour une itération principale pendant l'optimisation. Généralement, il est nécessaire de résoudre le système ou de le prévoir plus d'une fois pour avancer à la prochaine itération principale; la prévision est meilleure dans ce cas-ci, parce qu'elle doit calculer seulement la sensibilité des valeurs propres une fois pour une itération principale de l'algorithme. Après, une analyse d'erreur des valeurs propres et des vecteurs propres prévus est faite en vue de limiter la portée de la prévision dans le processus d'optimisation. L'analyse est faite pendant la maximisation d'espace entre deux valeurs propres adjacentes sur les systèmes structuraux et couplés de fluide-structure, modifiant une certaine variable structurale géométrique précédemment définie du modèle fini d'élément.

1. INTRODUCTION

It is common to find cases where two or more systems interact with one another. Those situations where it is not realistic to model each system independently of the others are known as coupled systems. Fluid-structure interactions belong to this class: neither the fluid domain nor the structural domain can be solved independently, as the forces at the interfaces exert a significant influence.

The problem of fluid-structure dynamic interaction is analyzed herein. It has applications in the analysis of sound transmission through the walls of pressure vessels, ducts, and vehicle cabins. Even though the displacements imposed on the fluid are assumed to be small, it is not possible to decouple the motions of the fluid and the solid.

It is inevitable that resonances will occur in such systems. These may reduce the sound transmission properties, and may even lead to structural failures. Thus it is desirable to identify these resonances, and, if possible minimize any adverse effects by re-designing the structure. It is during the re-design phase that optimization is employed.

Problems involving incompressible fluids are commonly referred to as hydro-elastic problems. Here the effects of fluid compressiblity are to be ignored, resulting in an elasto-acoustic problem. The systems are assumed to suffer but small perturbations about stable equilibrium points. This renders the governing equations in the fluid to be acoustic in nature and structure is considered to be a linear elastic solid.

Within this frame work it is important to chose appropriate variables that describe the system response. In the fluid domain displacement, pressure, velocity potential, or a combination thereof can be used. Here the non-symmetric ***u-p*** formulation and a finite element method have been chosen. Here ***u*** is the displacement of the structure and ***p*** is the pressure in the fluid. This choice is appropriate for the types of systems analyzed below.

Deneuvy [1] was one of the first to study coupled systems with a view to optimizing certain dynamic parameters. The goal was to design an optimal structure where separation of two adjacent natural frequencies was the design objective. One of the difficulties encountered was the choice of an appropriate convergence parameter that was needed to stabilize the optimization scheme.

More recently Pal and Hagiwara [2, 3] studied the optimization of noise level reduction in a coupled structural-acoustic problem. The objective was to minimize the changes in the design parameters to reach a predetermined response. Their method could only deal with those cases where the acoustic and structural resonant frequencies of the systems matched.

2. OBJECTIVE

Optimization problems based on dynamic criteria make use of the system eigenvalues, which in turn depend on the design variables. The optimization process requires that the objective function be calculated repeatedly. This re-computation is time-consuming for most systems. It would be desirable to be able to predict the eigenvalues for the new set of design parameters that are being identified during each loop of the iteration. The objective of this work is to determine the suitability and advantages of eigenvalue prediction.

The advantages of the proposed scheme is judged by the match of the predicted eigenvalues and eigenvectors with those derived by direct computation. Also, there should be a computational savings in terms of the number of floating point operations *–flops-* . *Flops* counting is a rather basic approach for evaluating the efficiency of a program or algorithm in as much as memory traffic and other operations associated with the operation of the code are not counted. Golub and Van Loan {4} argue that *flops* counting is a simple, but inexact accounting method that captures but one of the many factors that influence the computational efficiency of a code. Nevertheless, we believe that *flops* counting is adequate to test the viability of the predictive method. Also, the *flops* counter is a convenient feature of the *Matlab* software that was used to perform the necessary computational analysis.

3. ANALYZED SYSTEMS

3.1 Structural system *SE3* – bi-fixed beam of circular cross section

The structural system *SE3*, as shown in Figure 1, consists of a bi-fixed beam where the elements have length L_i and circular cross sections of inertia moment I_i. Possible control variables are the cross sections areas of the elements, A_i, or their diameters, ϕ_i. The structure has Young's modulus E and density ρ_s. L_T is the total length of the beam.

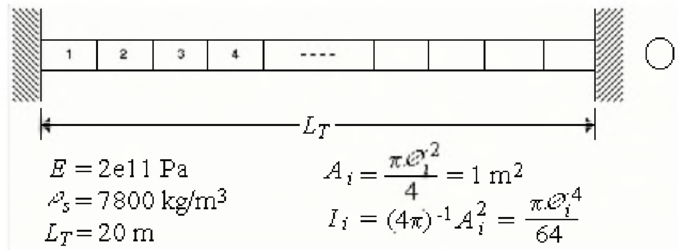

Figure 1. Structural system *SE3*

3.2 Structural system *SE4* - bi-fixed beam of rectangular cross section

This system is a bi-fixed beam under flexure with rectangular cross section of unitary width, b, as shown in Figure 2. The beam is modeled with elements of length L_i and cross sections of inertia moment I_i. Possible design variables are the cross sections areas of the elements, A_i, or their heights, e_i. It is observed that the mass matrix varies linearly and the rigidity matrix varies with the cube of the height, e.

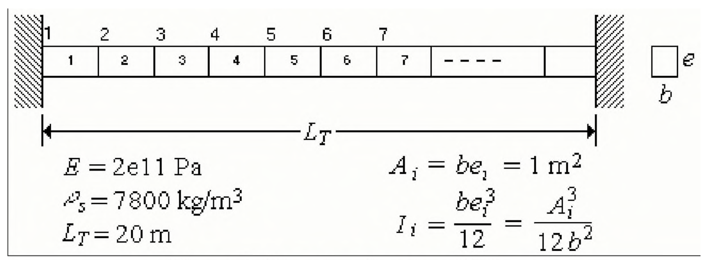

Figure 2. Structural system *SE4*

The structure has Young modulus E and density ρ_s, and the total length of the beam is L_T. Structural system *SE4* is classified as being of order 3, due to the exponent of the relation between the inertia moment and the area, $I=A^3/12$, for the unitary width.

3.3 Fluid-structure system *SFE1* - reservoir

The fluid-structure coupled system consists of a rectangular two-dimensional acoustic cavity of H=40m height and L_T=20m length, as shown in Figure 3. This model was presented previously by Olson and Bathe [5], Grosh and Pinsky [6] and Sandberg [7] among others; being a classical example where the basic phenomenon of the fluid-structure coupling can be evidenced. Boundary conditions are rigid sidewalls (R.W.) and free surface (F.S.) at the top; while the bottom side is modeled as a bi-fixed beam of rectangular cross section in flexure and unitary width, initially of square shape with uniform height of 1 m.

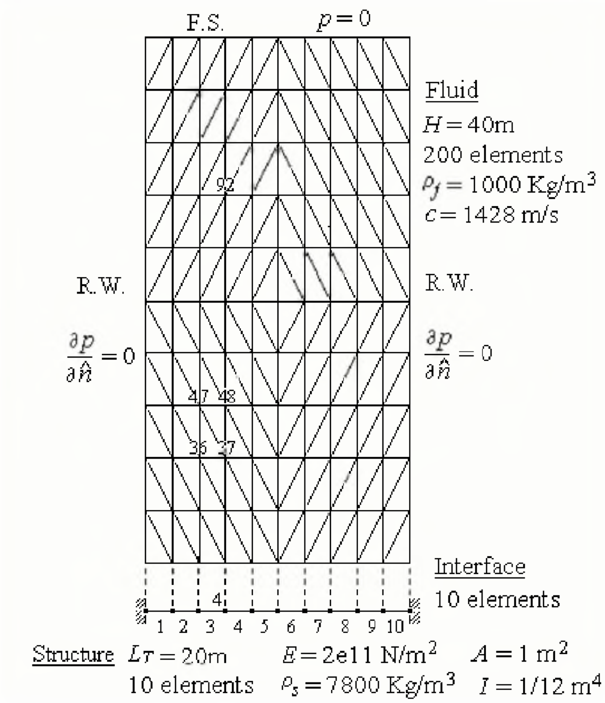

Figure 3. Fluid-structure system *SFE1*

Design variables are the heights of the beam elements, although the areas of the cross sections can also be used. The system is classified as being of order 3, due to the exponent of the relation between the inertia moment and the area of the structural cross section, $I=A^3/12$.

3. PERFORMANCE VERIFICATION OF THE *PREDICTIVE* FORMULAS

The sequential quadratic programming algorithm, implemented in the commercial software Matlab®, was used in this work, supplying the analytical expressions of the gradients of the objective function and the restrictions.

For verifying the numerical performance of the predictive formulas, regarding the number of float point operations, the fluid-structure coupled system *SFE1* was studied, choosing as design variables the heights of the structural elements which had a variation of up to 15%.

Figure 4 shows the quantity of flops and analyzed modes for solving the eigenvalues and eigenvectors problem just once, using both *solution* and *predictive* processes. It is observed fewer flops if the *predictive* option is used for few modes.

The *solution* process uses the *sptarn©* function supplied with the *Matlab® Partial Differential Equation Toolbox©*. The *sptarn©* function solves problems of generalized eigenvalues of the $(\boldsymbol{A}-\lambda\boldsymbol{B})\boldsymbol{x}=0$ system in the $[lb,ub]$ interval, where $\boldsymbol{A}$ and $\boldsymbol{B}$ are sparse matrices, $\boldsymbol{x}$ is the vector of independent variables, lb and ub are lower and upper limits of the searched eigenvalues. The *sptarn©* function uses the Lanczos method initially with $jmax$=100 base vectors, requiring a $jmax$*DOF workspace where DOF is the number of degree of freedom of the system.

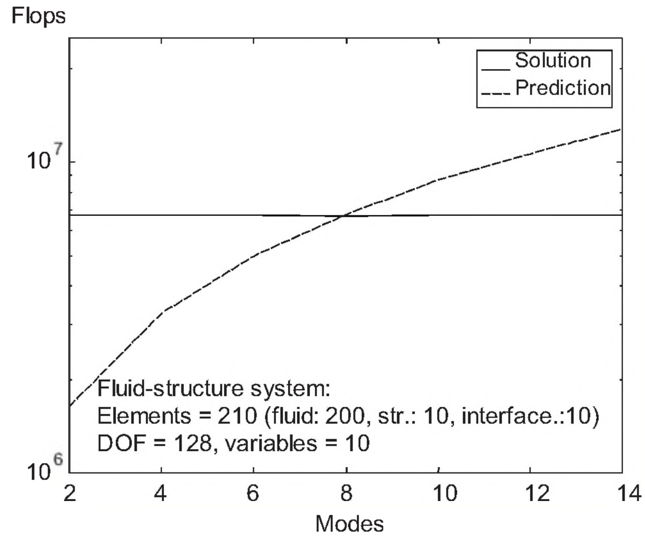

Figure 4. Flops with modes for solving the *SFE1* system once

Commonly, the algorithm stops when a sufficient number of eigenvalues converge; nevertheless, as the number of base vectors was maintained constant throughout the process, the quantity of flops in the interval varied little (Figure 4).

The quantity of flops, when the system is solved twice for a main iteration of the optimization, using the *solution* and *predictive* processes, is shown in Figure 5. In this case, the quantity of flops is fewer with the *predictive* option for all analyzed modes, which justifies its use for optimization of these systems, where many cycles must be performed for any iteration.

From these results it can be concluded that when it is used the predictive formulas in coupled fluid-structure systems, more efficient algorithms can be obtained regarding its computational cost. However, special techniques for solving the eigenvalues and eigenvectors problem can lead to situations more favorable to the *solution* process [7].

4. ACCURACY EVALUATION OF THE *PREDICTIVE* FORMULAS

An error analysis is carried out for the predicted eigenvalues, these calculated with the Rayleigh quotient method of Equation (1). Other error analysis is carried out for the predicted eigenvectors, these calculated with the finite difference method of Equations (2) and (3). The analyses are realized as a function of the allowable variation of the design variables. The aim of this study is to verify the validity of the *predictive* formulas, in such a way that the optimization processes can adequately converge.

$$\lambda_j^* = \frac{\overline{\boldsymbol{\phi}}_j^{(DF)^T} \boldsymbol{K}^* \boldsymbol{\phi}_j^{(DF)^T}}{\overline{\boldsymbol{\phi}}_j^{(DF)^T} \boldsymbol{M}^* \boldsymbol{\phi}_j^{(DF)^T}} \equiv \lambda_j^{(R)} \tag{1}$$

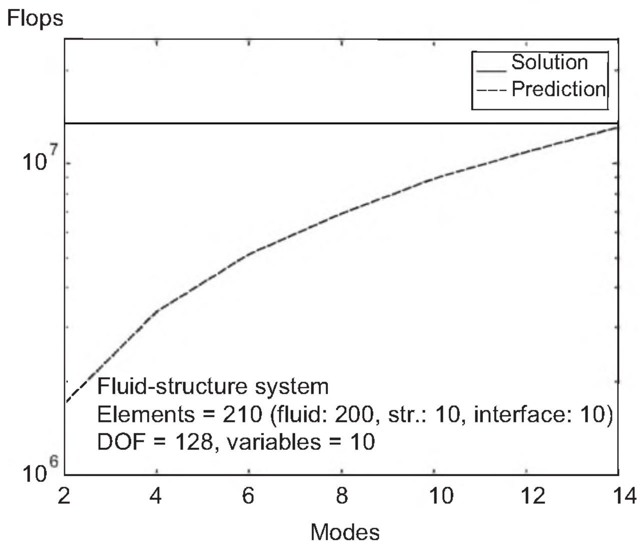

Figure 5. Flops with modes for solving the *SFE1* system twice

$$\boldsymbol{\phi}_j^* \approx \boldsymbol{\phi}_j + \boldsymbol{\phi}_j' \Delta e \equiv \boldsymbol{\phi}_j^{(DF)} \tag{2}$$

$$\overline{\boldsymbol{\phi}}_j^* \approx \overline{\boldsymbol{\phi}}_j + \overline{\boldsymbol{\phi}}_j' \Delta e \equiv \overline{\boldsymbol{\phi}}_j^{(DF)} \tag{3}$$

λ_j^* is the j^{th} eigenvalue and $\lambda_j^{(R)}$ is the j^{th} predicted eigenvalue of the modified system, ***K**** and ***M**** are the modified rigidity and mass matrices, $\overline{\boldsymbol{\phi}}_j^{(DF)}$ and $\boldsymbol{\phi}_j^{(DF)}$ are the j^{th} left and the j^{th} rigth predicted eigenvector of the modified system using the finite difference method, $\overline{\boldsymbol{\phi}}_j^*$ and $\boldsymbol{\phi}_j^*$ are the j^{th} left and the j^{th} right eigenvector of the modified system, $\overline{\boldsymbol{\phi}}_j^{(DF)}$ and $\boldsymbol{\phi}_j^{(DF)}$ are the j^{th} left and the j^{th} right predicted eigenvector of the modified system calculated with the finite difference method, $\overline{\boldsymbol{\phi}}_j$ .and $\boldsymbol{\phi}_j$ are the j^{th} left and the j^{th} right eigenvector of the coupled system, $\overline{\boldsymbol{\phi}}_j'$ .and $\boldsymbol{\phi}_j'$ are the derivatives of the j^{th} left and the j^{th} right eigenvector of the coupled system in relation to the structural variable e, and Δe is the variation of the structural height.

In order calculate the eigenvalues error, it was necessary to place in-phase the eigenvectors obtained by the *predictive* process, $\boldsymbol{\phi}_{prediction}$, in relation to the eigenvalues obtained by the *solution* process, $\boldsymbol{\phi}_{solution}$, according to Equation (4).

$$\frac{\boldsymbol{\phi}_{solution}^T \boldsymbol{\phi}_{prediction}}{\boldsymbol{\phi}_{solution}^T \boldsymbol{\phi}_{solution}} = \begin{cases} < 0, \ \boldsymbol{\phi}_{prediction} = -\boldsymbol{\phi}_{prediction} \\ > 0, \boldsymbol{\phi}_{prediction} = \boldsymbol{\phi}_{predicition} \end{cases} \tag{4}$$

For evaluating the error of the predicted eigenvector, erro$\boldsymbol{\phi}_{prediction}$, it was used the Euclidian norm that defines the error as,

$$\text{erro}\boldsymbol{\phi}_{prediction} = \left\| \boldsymbol{\phi}_{prediction} - \boldsymbol{\phi}_{solution} \right\| \tag{5}$$

First, the structural system *SE3* was analyzed, where the beam was discretized in 20 elements, which means 30 DOF. The system variables are the areas of the elements with a random variation between specified intervals, keeping unchanged the initial volume and the symmetry of the beam.

Figure 6 shows a maximum error of 0.96% in the prediction of the first ten frequencies, value found for a simultaneous variation of the areas of up to 25-30%. This error is smallest than the maximum error of approximately 5% obtained by Fox and Kaapor [8], who only studied the first three frequencies of a fixed-free beam of circular cross section, with a diameter variation of up to 30%.

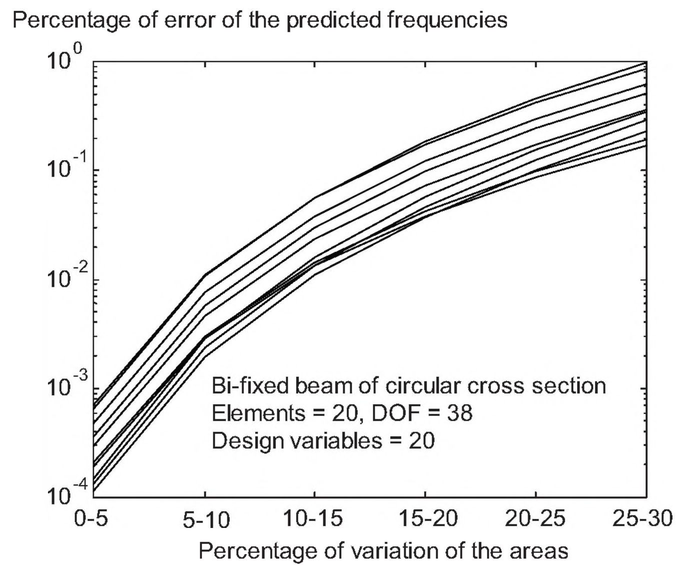

Figure 6. Prediction error of the first ten frequencies of the *SE3* system

The curves in Figures 6 to 11 are not labelled because the principal interest is to analyze the maximum error of the first ten predicted eigenvector and eigenvalues. Additionally, it is observed that the errors of the first frequencies do not correspond necessarily with the lower curves of the graphs.

Figure 7 shows a maximum error of 10.48% in the prediction of the first ten modes of the system *SE3*, taking a variation of the design variables of up to 25-30%. It is observed that for a variation of up to 10-15%, the maximum error is 2.45%, which is acceptable for optimization terms.

For obtaining major conclusions about modal error of the prediction, the *SE4* system was studied. The bi-fixed beam is discretized in 20 elements, producing a model with 38 DOF. The system variables are the heights of the elements, with a random variation between specified intervals, keeping unchanged the initial volume and the symmetry of the beam with a unitary width.

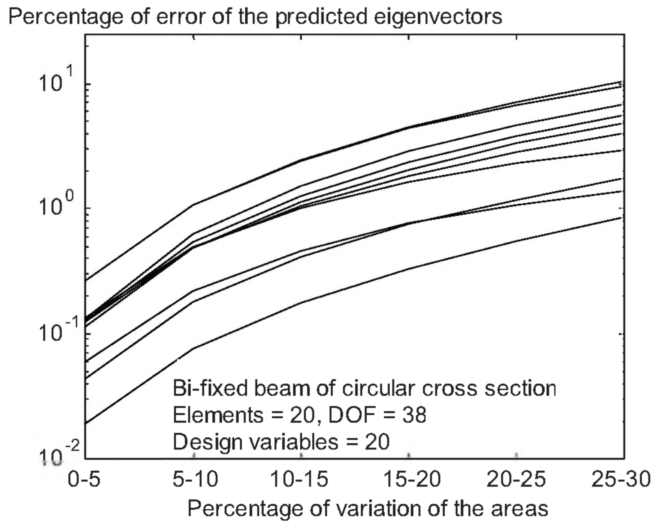

Figure 7. Prediction error of the first ten eigenvectors of the *SE3* system

Figure 8 shows a maximum error of 7.28% in the prediction of the first ten eigenvalues, for a simultaneous variation of the variables of up to 25-30%. This value is higher than the 0.96% of the second order *SE3* system, and higher than the maximum error of about 5% obtained by Fox and Kapoor (1968). This result shows the error increasing as a function of the non-linearity order given by the exponent of the relation between the inertia moment and the area, $I=kA^n$.

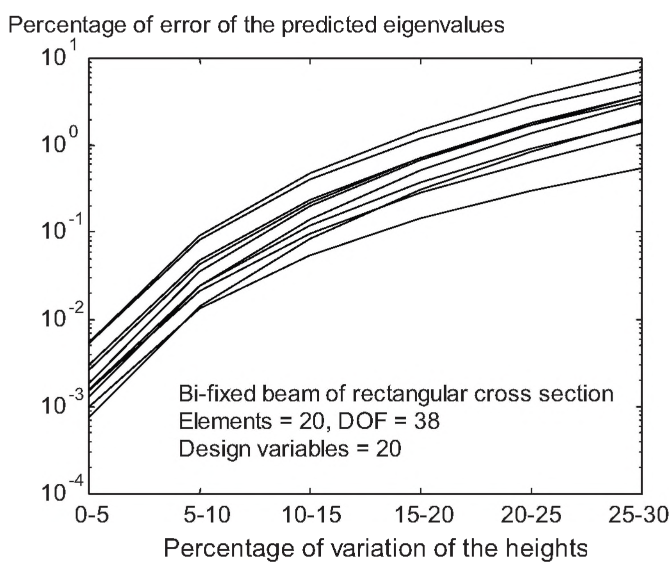

Figure 8. Prediction error of the first ten eigenvalues of the *SE4* system

Figure 9 indicates a maximum error of 30.58% in the prediction of the first ten eigenvectors, for a variation of the design variables of up to 25-30%. Moreover, it is observed that up to a 10-15% variation, the maximum error was 7.27%, value that could be high for the optimization, but it is important to remind that in practice the variables do not vary simultaneously in the same way.

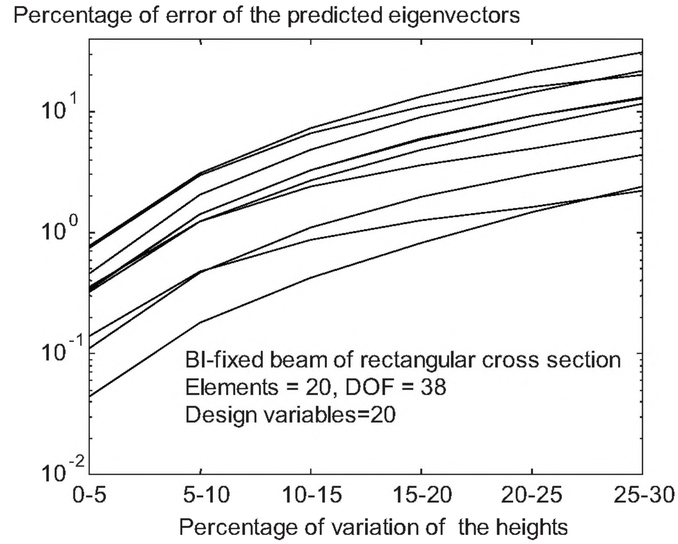

Figure 9. Prediction error of the first ten eigenvectors of the *SE4* system

Finally, the error of the modal prediction of the third order *SFE1* system is studied with the aim to establish conclusions on the prediction of eigenvalues and eigenvectors in coupled systems. The variables of the system were the heights of the elements, whose variation were made randomly in the intervals previously specified, maintaining the initial volume and the symmetry of the beam with an unitary width. It is observed that the order of the exponent of the relation between the moment of inertia and the area is three, identical to the previously analyzed case.

It is observed, from Figure 10, a maximum error of 0.42% in the prediction of the ten first frequencies for a simultaneous variation of the variables of up to 25-30%, value sufficiently lower than the maximum error of 7.28% of the *SE4* structural system. Some explanation originates by the fact that the error of the six fluid predominant frequencies must present a low value, because they vary little when the structural heights are modified. On the other hand, for the structural predominant coupled frequencies, i.e. frequencies 2nd and 3rd, the maximum error is lesser for the coupled case compared with the structural case of the system *SE4*.

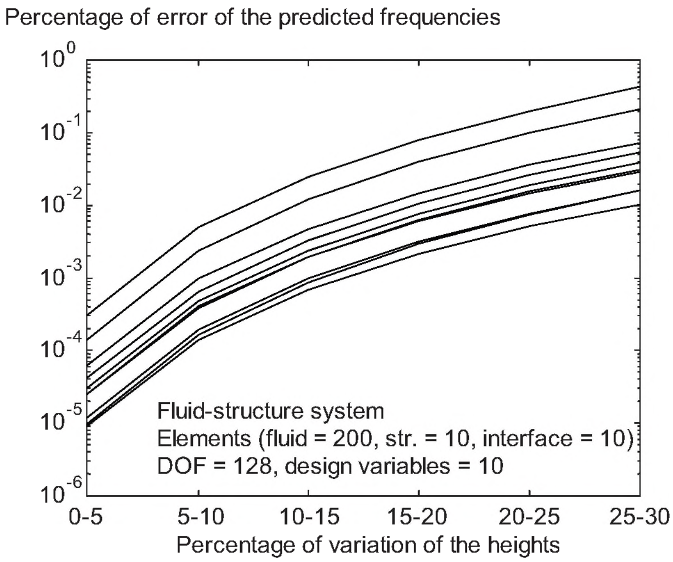

Figure 10. Prediction error of the first ten frequencies of the *SFE1* system

Figure 11 shows a maximum error of 9.60% in the prediction of the first ten natural modes, for a variation of the heights of up to 25-30%. This value is lower than the maximum error of 30.58% for the *SE4* structural system. It is also observed that for a variation of the variables of up to 10-15%, the maximum error of the predicted eigenvectors was 2.38%, which is lower than the maximum error of 7.27% in the system *SE4*.

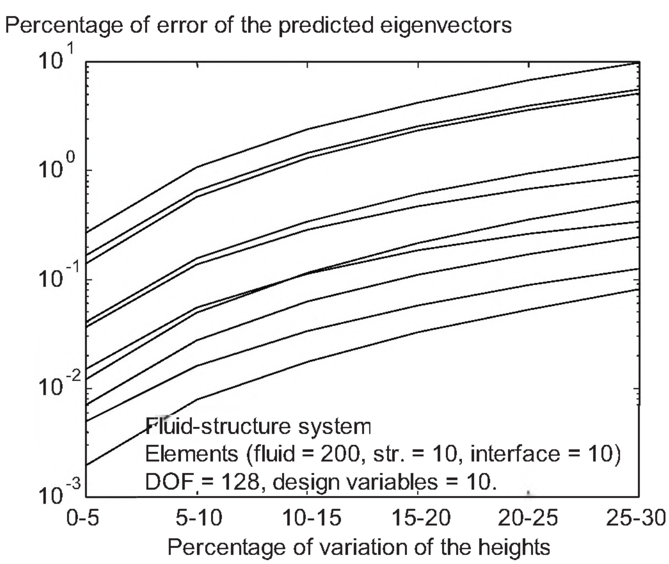

Figure 11. Prediction error of the first ten eigenvectors of the *SFE1* system

5. CONCLUSIONS

A methodology of using predictions for eigenvalues and eigenvectors has been presented. The formalism has been applied to a coupled fluid-structure system with the aim of optimizing the separation of two adjacent frequencies. The eigenvalues are predicted using the Rayleigh quotient and the eigenvectors are predicted with the aid of a finite difference scheme. The prediction formulas are restrained by certain conditions during the optimization process. These are in the form of the maximum allowable variation of the design variables.

The results suggest that the method is suitable for the optimization of structural and coupled fluid-structure optimization problems. Care must be taken to constrain the maximum variation of the design variables to values no greater than 10-15%.

6. REFERENCES

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