DATA COVARIANCE ESTIMATION FOR GEOACOUSTIC INVERSION

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1. INTRODUCTION

Estimating seabed geoacoustic parameters from ocean acoustic fields measured at an array of sensors represents a challenging but important nonlinear inverse problem. In a Bayesian approach, the posterior probability density (PPD) of the unknown geoacoustic model parameters is formulated in terms of the data information (represented by an appropriate likelihood function) and independent prior information [1, 2]. PPD properties such maximum *a posteriori* (MAP) model estimates (i.e., the most probable parameters) and marginal probability distributions for each parameter can be computed numerically using global optimization [3] and Markov-chain Monte Carlo integration methods [1, 2], respectively.

The likelihood function represents the conditional data uncertainty distribution interpreted as a function of the model parameters for the (fixed) measured data. Hence, specifying data uncertainties is an important component of Bayesian inversion. Data uncertainties must include both measurement error (e.g., errors due to instrumentation and ambient noise) and theory errors (due to the simplified seabed parameterization and idealized treatment of the forward problem). Theory errors, in particular, are difficult to estimate independently, and in most cases physically reasonable assumptions are required about the form of the uncertainty distribution. To date, data errors have been assumed to be Gaussian distributed and spatially uncorrelated (i.e., represented by a diagonal covariance matrix). However, the assumption of uncorrelated errors is often not valid due to theory errors. Neglecting significant error correlations represents the data as more informative than they actually are, and leads to under-estimating parameter uncertainties. In this paper, data-error correlations are estimated to form a full covariance matrix which is explicitly incorporated into the inversion procedure.

2. THEORY

Matched-field geoacoustic inversion is based on estimating a model $\mathbf{m}$ of seabed geoacoustic parameters by matching complex (frequency-domain) acoustic pressure fields $\mathbf{d}_f$ measured at an N-sensor array at f=1,F frequencies. Assuming the data errors are complex Gaussian distributed random variables uncorrelated from frequency to frequency but spatially correlated with covariance matrix $\mathbf{C}_f$ at the f-th frequency, the likelihood function is given by

$$L(\mathbf{m}) \propto \prod_{f=1}^{F} \frac{1}{|\mathbf{C}_f|} \exp[-\mathbf{r}_f(\mathbf{m})^T \mathbf{C}_f^{-1} \mathbf{r}_f(\mathbf{m})], \qquad (1)$$

where $\mathbf{r}_f(\mathbf{m}) = \mathbf{d}_f - \mathbf{d}_f(\mathbf{m})$ are data residuals (difference between measured and modelled data) and ^{T} represents conjugate transpose. The MAP model $\hat{\mathbf{m}}$ is computed by minimizing a mismatch function consisting of the negative log-likelihood plus a term representing prior information.

Under the usual assumption of spatially uncorrelated errors, the covariance matrix is diagonal, $\mathbf{C}_f = \sigma_f^2 \mathbf{I}$. In this case, the MAP estimate for the standard deviation is $\hat{\sigma}_f^2 = |\mathbf{r}_f(\hat{\mathbf{m}})|^2 / N$. However, if significant error correlations exist, the diagonal approximation is inadequate. Assuming the data residuals represent an ergodic random process, a non-parametric estimate of the full covariance matrix is given by

$$\hat{C}_{ij} = \sum_{k=1}^{N-|i-j|} [r_k(\hat{\mathbf{m}}) - \bar{r}(\hat{\mathbf{m}})]^* \, [r_{k+|i-j|}(\hat{\mathbf{m}}) - \bar{r}(\hat{\mathbf{m}})] / N, \qquad (2)$$

where $\bar{r}$ represents the residual mean and the subscript f is suppressed for clarity. Covariance elements that are located far off the main diagonal represent error correlations between widely spaced data points. These are expected to be small and are often poorly estimated due to the small number of samples in the average. Hence, it is generally beneficial to damp off-diagonal terms, e.g., by applying a cosine damping function. Since the MAP model is required to estimate the covariance but the covariance is itself required to estimate the MAP model (in the log-likelihood function), the above procedure must be applied iteratively.

The validity of the covariance estimate can be examined *a posteriori* by considering standardized residuals

$$\mathbf{w}_f(\hat{\mathbf{m}}) = [\hat{\mathbf{C}}_f^{-1/2}]^T \mathbf{r}_f(\hat{\mathbf{m}}), \qquad (3)$$

where $\hat{\mathbf{C}}_f^{-1/2}$ represents the inverse of the Cholesky decomposition (square root) of the covariance matrix. If the covariance estimate is valid, $\mathbf{w}_f$ should represent an uncorrelated random process: this can be examined qualitatively by plotting the autocorrelation of $\mathbf{w}_f$, with a narrow central peak indicating uncorrelated residuals. Quantitative statistical tests can also be applied to the standardized residuals (e.g., one-tailed runs test).

Finally, matched-field methods are typically employed without knowledge of the (complex) source spectrum. In this case, a maximum-likelihood estimate for the source strength can be derived [1, 2], leading to the substitution

$$\mathbf{d}_f(\mathbf{m}) \rightarrow \frac{\mathbf{d}_f(\mathbf{m})^T \mathbf{d}_f}{|\mathbf{d}_f(\mathbf{m})|^2}\mathbf{d}_f(\mathbf{m}) \qquad (4)$$

in the above equations.

3. INVERSION EXAMPLE

Bayesian geoacoustic inversion with full-covariance estimation is illustrated here for ocean acoustic data measured in the Mediterranean Sea off the west coast of Italy near Elba Island [3]. Linear frequency-modulated signals (300–800 Hz) were transmitted from a ship-towed source at approximately 10-m depth and recorded at a 48-element vertical line array that extended from 26–120 m depth in water 132-m deep. The inversion here is for a source transmission at a range of approximately 4 km. The seabed was parameterized as a two-layer model with an upper sediment layer of thickness h and sound speed c_1, density ρ_1 and attenuation α_1 overlaying a semi-infinite basement with parameters c_2, ρ_2 and α_2. Small corrections to the source range and depth and water depth were also included in the inversion, but are not discussed here.

Fig. 1 shows examples of the (complex) covariance matrices estimated from the acoustic data, and indicates that the spatial correlation scale decreases with frequency. Fig. 2 shows marginal PPDs for the geoacoustic parameters computed using both full covariance-matrix estimates and variance-only estimates. The uncertainty distributions based on variance-only estimates are overly optimistic for all geoacoustic parameters compared to the full-covariance estimates, in some cases indicating unrealistic parameter sensitivity (e.g., α_1, α_2, ρ_2). Finally, Fig. 3 shows that incorporating full covariance matrices in the inversion leads to essentially uncorrelated standardized residuals, indicating that the error correlations have been accounted for correctly in the inversion.

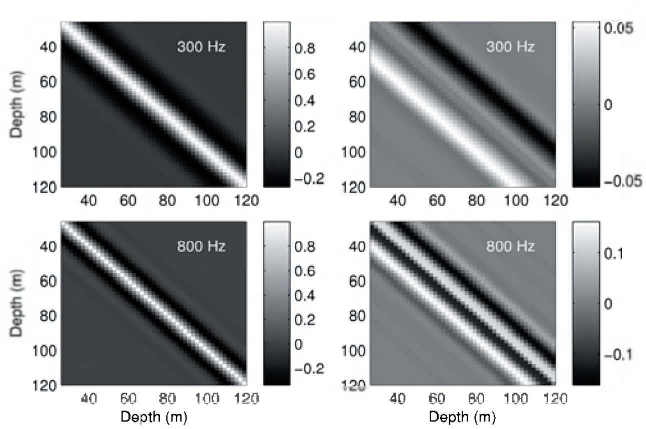

Fig. 1. Covariance matrices at 300 and 800 Hz (real part on left, imaginary part on right), normalized by amplitude of real part.

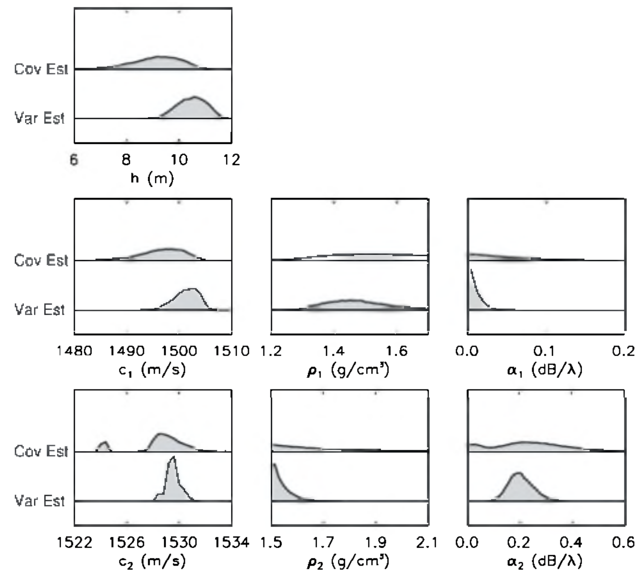

Fig. 2. Marginal probability distributions for geoacoustic parameters computed using full covariance matrix estimates (Cov Est) and variance-only estimates (Var Est).

REFERENCES

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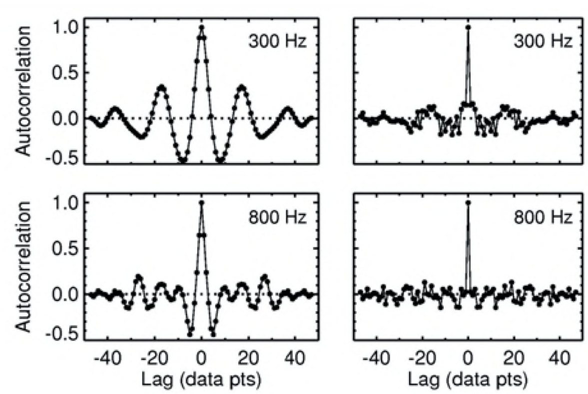

Fig.3. Autocorrelation of standardized residuals (real part) for variance only estimates (left) and full covariance estimates (right).