Hearing Aid Modeling with Adaptive Nonlinear Filters

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1. Introduction

In this paper, the performance of nonlinear adaptive filters in modeling devices with mild and moderately strong nonlinearities such as hearing aids is investigated. The Volterra series based nonlinear adaptive filter structure and bilinear adaptive filter structure are compared with their linear counterparts of equal number of tap weights. Experiments for two types of hearing aids using human voice sequences are carried out in a system identification fashion. The number of tap weights used range from 200 to 800 and the adaptation is done in normalized least mean squares (NLMS) form (Haykin 2002). Few graphical results of system identification with white noise input is shown for both Volterra and bilinear cases. The use of such filters for hearing aid modeling is then discussed. The results on the improvement attainable using nonlinear filters are presented. They verify the existence of nonlinearities in hearing aids and that nonlinear adaptive filters are better suited for hearing aid modeling compared to their linear counterparts with equal number of filter coefficients. The signal to noise ratio improvement in modeling, shows improvements in the range of 5% to 50%. The problem dependent but important aspect of the careful selection of the number of linear weights (coefficients of linear terms) and the number of nonlinear coefficients (coefficients of the nonlinear terms) is discussed.

2. Nonlinear Filters

While linear filters are predominantly used in practice, there are many applications where nonlinear filters are required. Truncated Volterra series expansion based filters and bilinear filters relate the input signal of a (nonlinear) system to the output using a polynomial model of nonlinearity (Mathews 1991). Bilinear filters reduce the large number of coefficients required in Volterra form by using a recursive nonlinear difference equation. Bilinear filters have been successfully used to reduce the saturation effects of active noise cancellation systems (Kuo & We 2005). Hearing aids are known to be nonlinear devices. Therefore, the use of nonlinear adaptive filters for the modeling of hearing aids should produce better results than linear adaptive filters.

2.1 Volterra Series Expansion

Let $x(n)$ and $y(n)$ represent the input and output signals respectively. The Volterra series expansion for $y(n)$ using $x(n)$ is given by

$$\begin{aligned} y(n) &= h_0 + \sum_{i_1=0}^{\infty} h_1(i_1)x(n-i_1) + \cdots \\ &+ \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\cdots\sum_{i_p=0}^{\infty}\big(h_p(i_1,i_2,\cdots,i_p) \\ &\qquad x(n-i_1)x(n-i_2)\cdots x(n-i_p)\big) \end{aligned} \tag{1}$$

where $h_p(i_1, i_2, \cdots, i_p)x(n-i_1)x(n-i_2)$ is called the p^{th} order Volterra kernel of the system.

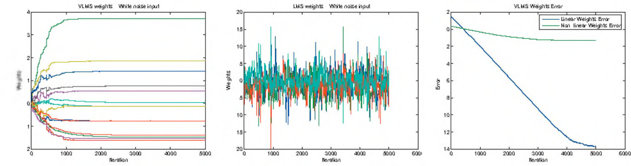
2.2 Bilinear Filters

Volterra series representation requires a large number of coefficients to handle higher order nonlinear systems. An alternative representation is the recursive nonlinear difference equation including the simple model characterizing bilinear filters with input–output relationship,

$$\begin{aligned} y(n) &= \sum_{i=1}^{N-1} c_i y(n-i) + \sum_{i=0}^{N-1}\sum_{j=1}^{N-1} b_{i,j} y(n-j)x(n-i) \\ &+ \sum_{i=0}^{N-1} a_i x(n-i) \end{aligned} \tag{2}$$

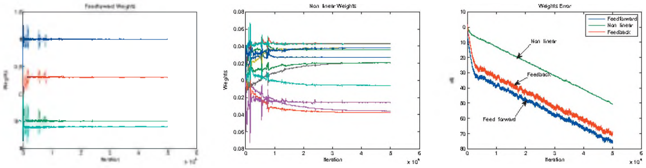
3. Hearing Aid Modeling

The performance of hearing aids are usually modeled using human voice sequences rather than white noise. A reference signal (an audio recording) is fed to the hearing aid while the output of the hearing aid is recorded. Consequently, The reference signal and the hearing aid output should be aligned over time to compensate for the delays involved. This can be done by simply locating the maximum of the cross-correlation. For the purpose of comparison, the non linear adaptive filter and a linear filter of equal length should be compared. If the number of samples considered (and therefore the number of linear coefficients) is N, the total number of coefficients is $N + N^2$. For example if a Volterra filter is compared with a linear filter with 240 taps, the Volterra filter can comprise of 15 linear coefficients and $15^2 = 225$ nonlinear coefficients. The small number of linear coefficients and the comparatively very large number of nonlinear coefficients are not desirable to model mild and slightly strong nonlinear filters. Therefore the coupling between the number of linear coefficients and the number of nonlinear coefficients is removed and filters with length $M_{li} + M_{nl}$ are compared. Here M_{nl}, the number of nonlinear coefficients, is a square number and $M_{nl} < M_{li}$, which is the number of linear coefficients. In the implementation, the latest $M = M_{li} + M_{nl}$ samples, $\mathbf{u}_{li}, \quad (1 \times M)$ are selected



(a) LMS (b) VLMS (c) error

Fig. 1. LMS weights, VLMS weights, VLMS weights error.



(a) FF (b) NL (c) error

Fig. 2. Bilinear filter performance. FF: feedforward, NL: nonlinear weights

and used with the linear filter. The first M_{li} samples of $\mathbf{u}_{li}$ and the mutual products of the first $M_{nl}^{1/2}$ of $\mathbf{u}_{li}$ are used to compose the input to the nonlinear filter $\mathbf{u}_{nl}, \quad (1 \times M)$.

4. Experiments and Results

4.1 Volterra and Bilinear Filters

A second order Volterra filter was implemented for system identification of a nonlinear system with the following Volterra kernels. Resulting plots are shown in Figure 1.

$$h1 = \begin{bmatrix} -0.78 & -1.48 & -1.39 & 0.04 \end{bmatrix}$$

$$h2 = \begin{bmatrix} 0.54 & 3.72 & 1.86 & -0.76 & 3.72 & -1.62 & 0.76 & -0.12 \\ 1.86 & 0.76 & 1.41 & -1.52 & -0.76 & -0.12 & -1.52 & -0.13 \end{bmatrix}$$

A bilinear filter was tested for a system identification problem for a system with the parameters a, the feedforward weights, b, the nonlinear weights, and c, the feedback weights. Resulting plots are shown in Figure 2.

$$a = \begin{bmatrix} 1.0 & -0.5 & 0.3 & -0.6 \end{bmatrix}^T$$

$$b = \begin{bmatrix} 0.027 & 0.036 & 0.043 & 0.042 & -0.036 & 0.038 \\ 0.021 & 0.038 & 0.020 & -0.038 & -0.006 & -0.026 \end{bmatrix}$$

$$c = \begin{bmatrix} 0.2 & -0.3 & 0.4 \end{bmatrix}^T$$

Here, b is shown in a matrix form instead of the vector for convenience. The entries in b were deliberately selected to be an order of magnitude smaller to ensure convergence.

4.2 Hearing Aid Modeling Results

The ability of linear and nonlinear filters of modeling hearing aids was compared for two different types of hearing aids: A with a mild nonlinearity and B with a moderately strong nonlinearity. The comparison measure used was the percentage SNR improvement gained by using the nonlinear filter as

$$\text{Improvement} = \frac{(S/N)_{nl} - (S/N)_{li}}{\left[(S/N)_{nl} + (S/N)_{li}\right]/2} \times 100\% \tag{3}$$

where $(S/N)_{nl}$ is the SNR between the desired signal and the modeling error due to the use of the nonlinear NLMS filter and $(S/N)_{li}$ is the same ratio for the linear NLMS filter.

Table 1 and Table 2 summarize the results for modeling hearing aid A and modeling hearing aid B, respectively, using Volterra series based non linear filters. M is the number of filter coefficients and M_{li} and M_{nl} are the linear and non-linear coefficients respectively. The convergence parameters μ and the run length (iterations) are also listed.

Table 1. Volterra based hearing aid modeling, SNR improvement for A

Length M	M_{li}	M_{nl}	μ	Run	Improvement
249	249	49	0.8	50000	4.6324%
249	249	49	0.2	50000	9.0757%
500	500	100	0.2	50000	12.3211%

Table 2. Volterra based hearing aid modeling, SNR improvement for B

Length M	M_{li}	M_{nl}	μ	Run	Improvement
249	249	49	0.8	50000	7.8183%
249	249	49	0.2	50000	6.1692%
500	500	100	0.2	50000	7.8698%
500	500	400	0.2	50000	55.7738%

Hearing aid modelling was done using bilinear filters and compared with the linear counterparts of equal length. Table 3 shows the results for modeling hearing aid B. M_{ff}, M_{fb} and M_{nl} are the number of feed forward coefficients, number of feedback coefficients and the number of nonlinear coefficients respectively. The run length was selected to be 10000. Bilinear filters being not as exact as Volterra filters in approximating the nonlinearity could be a reason for weaker performance.

5. Discussion

Nonlinear adaptive filters perform significantly better than their linear counterparts, in approximating mild and moderately strong nonlinear hearing aids. As the nonlinearity of the device becomes stronger, a larger number of nonlinear coefficients should be used. The SNR improvement in modeling, showed improvements in the range of 5% to 50%. The match between the number of linear and the mumbler of nonlinear coefficients should be selected to suit the system in question. The SNR seems to vary depending on the above coefficient numbers and the values of convergence parameters selected. These effects should further be investigated.

References

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Kuo, S. M. & We, H.-T. (2005), 'Nonlinear adaptive bilinear filters for active noise control systems', *IEEE Transactions on Circuits and Systems I* **52**(3), 617–624.

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Table 3. Bilinear hearing aid modeling, SNR improvement for A

M	M_{ff}	M_{fb}	M_{nl}	μ_a	μ_b	μ_c	Impr.
205	103	102	0	0.2	0.005	0.01	0.88553%
205	100	99	6	0.2	0.005	0.01	
1379	690	689	0	0.2	0.005	0.01	2.4973%
1379	500	499	380	0.2	0.005	0.01	
809	405	404	0	0.2	0.005	0.01	7.9348%
809	300	299	210	0.2	0.005	0.01	