# FINITE-ELEMENT MODELLING OF ACOUSTIC WAVE SCATTERING FROM FLUID, RIGID AND ELASTIC OBJECTS

**Omar Falou[1], J. Carl Kumaradas[1,2], and Michael C. Kolios[1,2]**
[1]Dept. of Electrical and Computer Engineering, Ryerson University, 350 Victoria St., ON, Canada, M5B 2K3
ofalou@ryerson.ca
[2]Dept. of Physics, Ryerson University, 350 Victoria St., ON, Canada, M5B 2K3

## 1. INTRODUCTION

Ultrasound has long been used as a diagnostic technique in which high frequency sound waves are directed into the body. Structures and tissues within the body reflect (scatter) the sound, and the resulting echoes are processed by a computer to produce an image. It has already been shown that high-frequency ultrasound (20MHz-60MHz) can be used to detect structural and physical changes in certain cell lines and cell ensembles during induced cell death, and in particular during apoptosis[1, 2].

In order to determine the proportion of responding cells in a tissue (to assess the effectiveness of the chemotherapy), a theoretical model of acoustic wave scattering by cell ensembles is required. The majority of ultrasound tissue models do not take into account the elastic nature of cells, which maybe significant at high frequencies. Therefore, a more complete model which incorporates the mechanical properties of cells is needed. The initial goal of this work is to develop a scattering model of a single cell.

Analytical solutions to the problem of wave scattering from three dimensional structures such as spheres have been studied extensively in the past. However, these formulations are not flexible enough to be extended to wave scattering from complex geometries such as those of cells. In this paper, we develop a finite element model of acoustic wave propagation through 3-D arbitrary-shaped structures to solve the problem of high-frequency acoustic scattering. The long term goal of this research is to understand the interaction of ultrasound with micrometer scale objects such as cells.

## 2. METHOD

The FEMLAB® software package (COMSOL AB, Stockholm) was used to develop and solve acoustic scattering models from fluid, rigid and elastic spheres immersed in a fluid medium, for which analytical solutions are available. The descriptions of these models are given in sections 2.1, 2.2 and 2.3 respectively.

### 2.1. Scattering from a Fluid Sphere

The entire computation domain is modelled as a sphere filled with fluid domains. A radiation (absorbing) boundary condition is applied on the domain boundary (surface of sphere). This boundary has a dual role: a) it approximates an infinite space b) it acts a wave source for an incident plane wave. The scatterer is modelled as a sphere centered in the middle of the domain. At the scatterer surface the particle normal acceleration is the same, thus a continuity boundary condition is required.

Inside and outside the spherical scatterer, wave propagation is governed by the Helmholtz equation of the acoustics application mode supplied by FEMLAB.

### 2.2. Scattering from a Fluid Immovable Sphere

The setup of this model is similar to that of the scattering from a fluid sphere described 2.1, except the fluid sphere is replaced by a rigid immovable one. In FEMLAB, this scenario is realized by disabling the acoustic wave equation (Helmholtz equation) inside the scatterer domain and by setting the scatter-surrounding medium interface boundary condition to “sound hard” condition which sets the acceleration at the surface to zero. This model used ~95,000 mesh elements and ~85,000 degrees of freedom.

### 2.3. Scattering from an Elastic Sphere

The two previous models assume only compressional waves. The Helmholtz equation is sufficient to describe this propagation. When a scatterer is elastic, in addition to compressional waves, shear and surface waves, may also exist inside and at the surface of the scatterer, respectively. In order to model all these phenomena, the FEMLAB 3D solid, stress-strain application mode is used inside the scatterer. The scatterer is surrounded by a fluid governed by the Helmholtz equation. Continuity of the normal component of acceleration is imposed as a boundary condition at the scatterer surface. ~93,000 mesh elements and ~44,000 degrees of freedom were used in this model.

## 3. RESULTS

In order to validate the three FEMLAB models, three incident wave frequencies (or ka’s) were used for each model.

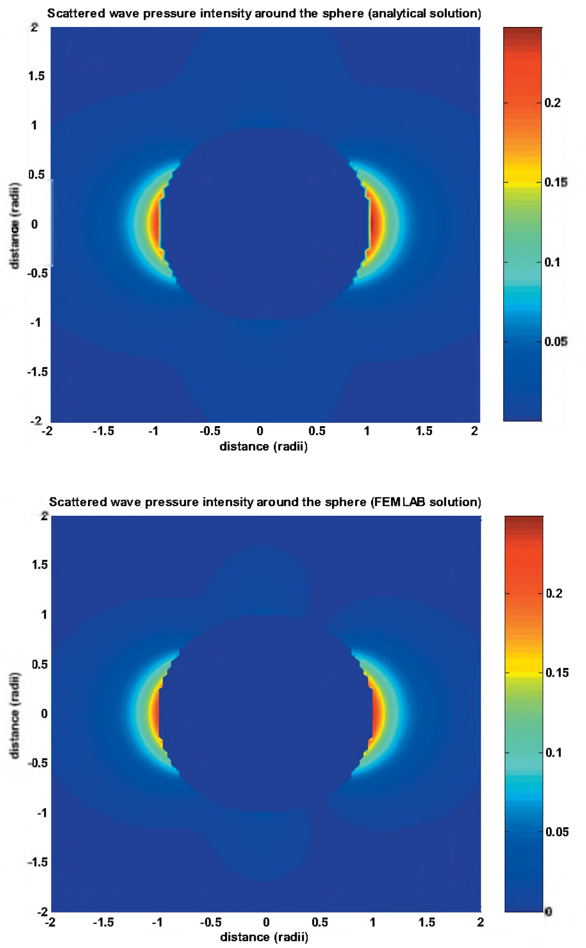


Fig. 1. Analytical (top) and FEMLAB (bottom) solution for the elastic scattering problem. The plots show the near-field pressure intensity for a slice parallel to the direction of wave propagation (left to right) through the origin. The solution inside the sphere was not calculated.

The results of these simulations were compared against analytical solutions derived by Anderson [3] (fluid scatterer) and Faran [4] (rigid immovable and elastic scatterer).

The scattered wave pressure intensity at 180 locations, situated at a distance at one and a half radii from the scatterer center was used to calculate the average error. Table 1 shows the calculated average error of the FEMLAB solution with respect to the analytical solution for each of the models.

Figure 1 compares the analytical and the FEMLAB solutions for an elastic sphere (polystyrene) immersed in water for ka = 1. It shows a slice parallel to the direction of propagation of the incident plane wave, which travels from left to right. Due to the cylindrically symmetric nature of the model, the plot is independent of its angle of rotation around the axis of propagation.

Table 1. Average error for each model

| Scatterer Type | ka | % Error |
|---|---|---|
| Low Contrast Fluid | 0.8378 | 1.43 |
| (density: 1060 kg/m$^3$ and | 1.0 | 1.47 |
| sound speed: 1530 m/s) | 3.0 | 12.08 |
| Rigid Immovable | 0.8378 | 7.30 |
| | 1.0 | 5.77 |
| | 3.0 | 7.99 |
| Elastic (Polystyrene) | 0.8378 | 2.05 |
| | 1.0 | 4.24 |
| | 3.0 | 6.42 |

## 4. DISCUSSION AND CONLUSION

The average error of the FEMLAB solution with respect to the analytical solution generally increases as ka increases. This could be explained by the variation of the mesh density with respect to the wavelength of the incident wave. A higher value of ka, implies a smaller wavelength, which leads to a lower mesh density per wavelength. A lower mesh density causes a decrease in the accuracy of the solution.

The finite-element models presented above provide accurate predictions of wave scattering by three types of scatterers, fluid, rigid immovable and elastic. These models can be extended to solve acoustic scattering problems from arbitrarily shaped and complex structures such as cells.

## REFERENCES


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