## **MODE SCANNING FOR ULTRASOUND PHASED ARRAYS WITH SIX PLANES OF SYMMETRY**

**Duo Chen, and Robert J. McGough**

Dept. of Electrical and Computer Engineering, Michigan State University, East Lansing, Michigan, 48824

## 1. **ABSTRACT**

Modes are symmetric focal patterns that exploit the symmetry of ultrasound phased arrays and cancel the complex pressure field in multiple planes that contain the central axis. The mode scanning approach, which was originally derived for symmetric planar arrays populated with rectangular elements, is extended to include triangular and hexagonal elements. This focusing method is very useful for generating heating patterns in hyperthermia. An example of mode scanning with 6-fold symmetry shown for a flat, two-dimensional ultrasound phased array populated with equilateral triangles. The pressure field of a single triangular element is computed with the fast nearfield method, which generates pressure fields with relatively small errors. The derivation of the element driving signals that produce different model focal patterns for any mode combines the characteristic of geometric symmetry with the method of gain maximization.

### **2. INTRODUCTION**

Various scanning techniques generate useful heating patterns for hyperthermia with ultrasound phased array applicators. Spot scanning, which is effective for small and superficial tumors, is a focusing approach that is often employed, but for large and deep tumors this method generally produces axial hot spots [1]. Another technique is mode scanning [2], which has been demonstrated using a square array that produces a four-focus mode. Mode scanning cancels axial pressure fields and therefore effectively heats larger tumors. This paper demonstrates the results of mode scanning with 6-fold symmetry shown for a flat, two-dimensional ultrasound phased array populated with equilateral triangles.

## **3. METHODS**

#### 3.1 Mode definition

Modes are symmetric focal patterns that exploit the symmetry of ultrasound phased arrays and cancel the complex pressure field in multiple planes that contain the central axis.

#### 3.2 Excitation scheme

The gain maximization method [3] generates useful driving signals for multiple focus patterns. The complex focal point pressures, p, relate to the complex column vector of array element surface particle velocities, u, through the matrix relation

 $p=Hu$ ,  $(1)$ 

where H is a complex forward propagation matrix that describes the transfer relationship between all array elements and all distinct focal points. With the gain maximization method, the normalized intensity gain,  $G=\langle p, p \rangle/\langle u, u \rangle$ , is maximized in order to minimize the total energy needed by the phased array to achieve given focal intensity values. For a single focus, a phase-only excitation scheme adopted from the gain maximization method computes the phase vector through the formula

$$
\theta = \arg(H^{*t}) + \arg(p), \ (2)
$$

and the amplitudes of the array elements are determined through the formula

$$
A = \frac{|p|}{\langle h, \exp(j\theta)\rangle}.\tag{3}
$$

#### 3.3 Mode derivation

The derivation of the element driving signals that produce different focal patterns for any mode combines the characteristic of geometric symmetry with the method of gain maximization. The symmetric element arrangement of the array corresponding to six foci is shown in Fig. 1. Using this ordering scheme, the partitioned complex excitation verctor can then be written as

$$
u = [\widetilde{u}^{\top} \widetilde{u}^{\top} \widetilde{u}^{\top} \widetilde{u}^{\top} \widetilde{u}^{\top} \widetilde{u}^{\top} \widetilde{u}^{\top}]^T
$$
  
=  $[\widetilde{u}^{\top} \widetilde{u}^{\top} e^{\prime \pi} \widetilde{u}^{\top} \widetilde{u}^{\top} e^{\prime \pi} \widetilde{u}^{\top} \widetilde{u}^{\top} e^{\prime \pi}]^T$  (4)

In order to match the partitioning of the input excitation **u,** the forward propagation vector can be separated as

$$
h = \left[\widetilde{h}^1 \; \widetilde{h}^2 \; \widetilde{h}^3 \; \widetilde{h}^4 \; \widetilde{h}^5 \; \widetilde{h}^6 \; \right], \qquad (5)
$$

so equation (1) can be rewritten as

$$
p = \widetilde{h}^1 \widetilde{u}^1 + \widetilde{h}^2 \widetilde{u}^2 + \widetilde{h}^3 \widetilde{u}^3 + \widetilde{h}^4 \widetilde{u}^4 + \widetilde{h}^5 \widetilde{u}^5 + \widetilde{h}^6 \widetilde{u}^6
$$
.(6)



*Fig. 1: Flat planar 2D ultrasound phased array comprised*  $of$  triangular pistons.

For six plane symmetric foci, the positions of the six focal points mimic the symmetry of the element ordering scheme. The six complex pressures are expressed as  $p_1$ ,  $p_2$ ,  $p_3$ ,  $p_{\mu}$ ,  $p_{\mu}$ , and  $p_{\mu}$ . All of these relationships are combined into a single matrix expression according to

$$
\begin{bmatrix} P_{1} \\ P_{2} \\ P_{3} \\ P_{4} \\ P_{5} \\ P_{6} \end{bmatrix} = \begin{bmatrix} \overline{\widetilde{h}}^{1} & \overline{\widetilde{h}}^{2} & \overline{\widetilde{h}}^{3} & \overline{\widetilde{h}}^{4} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{6} \\ \overline{\widetilde{h}}^{2} & \overline{\widetilde{h}}^{1} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{4} & \overline{\widetilde{h}}^{6} \\ \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{4} & \overline{\widetilde{h}}^{6} \\ \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{6} \\ \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} \\ \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} \\ \overline{\widetilde{h}}^{6} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} & \overline{\widetilde{h}}^{5} \end{bmatrix} \begin{bmatrix} \widetilde{u}^{1} \\ \widetilde{u}^{2} \\ \widetilde{u}^{4} \\ \widetilde{u}^{6} \end{bmatrix}.
$$

When the amplitude at each focus is equal, that is,  $|p_1| = |p_2|$  $= |p_3| = |p_4| = |p_5| = |p_6|$ , then

$$
p_{\scriptscriptstyle{1}} = p_{\scriptscriptstyle{4}} e^{\scriptscriptstyle{i}\prime\prime} = p_{\scriptscriptstyle{3}} = p_{\scriptscriptstyle{4}} e^{\scriptscriptstyle{i}\prime\prime} = p_{\scriptscriptstyle{5}} = p_{\scriptscriptstyle{6}} e^{\scriptscriptstyle{i}\prime\prime}
$$

$$
= [\widetilde{h}^{\scriptscriptstyle{1}} + e^{\scriptscriptstyle{i}\prime\prime} \widetilde{h}^{\scriptscriptstyle{2}} + \widetilde{h}^{\scriptscriptstyle{3}} + e^{\scriptscriptstyle{i}\prime\prime} \widetilde{h}^{\scriptscriptstyle{4}} + \widetilde{h}^{\scriptscriptstyle{5}} + e^{\scriptscriptstyle{i}\prime\prime} \widetilde{h}^{\scriptscriptstyle{6}}] \widetilde{u}^{\scriptscriptstyle{1}} \cdot (8)
$$

Thus, the forward propagation matrix H of a six focus mode can be reduced to a single vector,

$$
\widetilde{h} = [\widetilde{h}^{\mathrm{T}} - \widetilde{h}^{\mathrm{T}} + \widetilde{h}^{\mathrm{T}} - \widetilde{h}^{\mathrm{T}} + \widetilde{h}^{\mathrm{T}} - \widetilde{h}^{\mathrm{T}}] \tag{9}
$$

In order to obtain the driving signals for the entire array, the *h* vector is computed first using equation (9), and then equation (2) computes the excitation  $\widetilde{\mathcal{U}}^1$ .

## **4. RESULTS**

Mode scanning simulations are evaluated for a flat, two-dimensional ultrasound phased array populated with 150 equilateral triangles. The length of each element is 1 wavelength. The pressure field of a single triangular element is computed with the fast nearfield method [4][5], which generates pressure fields with relatively small errors.

Figure 2 shows the pressure field generated by this array of triangular sources in the focal plane. The focal plane is 80 wavelengths from the surface of the phased array. Fig. 2 contains six symmetric foci and three planes of symmetry in which the pressure field cancels perfectly.



*Fig. 2: Pressure field computed in the focal plane with mode scanning.*

# **5. CONCLUSION**

These results show that, with the combination of geometric symmetry and the method of gain maximization, element driving signals can be selected to generate sysmmetric patterns that cancel the pressure field in symmetric planes that intersect along the array normal and therefore eliminate axial hot spots.

## **REFERENCES**

[1] Moros E., Roemer R. and Hynynen, K., 1990, Pre-focal plane high-temperature regions induced by scanning focused ultrasound beams. *International Journal of Hyperthermia*, 6, 351-366.

[2] McGough, R. J., Wang, H., Ebbini, E. S. and Cain, C. A. 1994. Mode scanning: heating pattern synthesis with ultrasound phased arrays. *International Journal of Hyperthermia*, 3, 433-442.

[3] Ebbini, E. S. and Cain, C. A., 1991, Optimization of the intensity gain of multiple-focus phased array heating patterns. *International Journal o f Hyperthermia,* 7, 953-973.

[4] McGough, R. J., Samulski. T. V., and Kelly, J. F. 2004. An efficient grid sectoring method for calculations of the nearfield pressure generated by a circular piston. *J. Acoust. Soc. Am.* 115 [5], 1942-1954.

[5] McGough, R. J. 2004, "Rapid calculations of time-harmonic nearfield pressures produced by rectangular pistons," *J. Acoust. Soc. Am.,* 115(5).