THE FAST NEARFIELD METHOD APPLIED TO AXISYMMETRIC RADIATORS

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1. INTRODUCTION

The pressure fields radiated by baffled circular pistons have been studied extensively by the acoustics community [1]. Since many realistic transducers have a velocity profile that varies in the radial direction, the field generated by axisymmetric radiators is an important application [2,3]. In an earlier work, the authors derived a fast nearfield method (FNM) for pistons that have a spatially uniform velocity [4]; and in this paper, the FNM is extended to radiators with a radially varying normal velocity. This fast nearfield method is implemented and compared with the standard Rayleigh integral approach.

2. THEORY

Consider a circular piston of radius *a* lying in the *x*-*y* plane surround by an infinite, rigid baffle. Assume a lossless, homogeneous medium with density ρ and sound speed *c*. The single-frequency pressure $P_a(r,z;\omega)$ generated by this piston with a spatially uniform velocity of unity is given by the fast nearfield method (FNM)

$$P_a(r,z;\omega) = \frac{\rho c a}{\pi}\int_0^{\pi} \frac{r\cos\psi - a}{r^2 + a^2 - 2ar\cos\psi} \times \left(e^{-jk\sqrt{r^2+a^2+z^2-2ar\cos\psi}} - e^{-jkz}\right) d\psi \tag{1}$$

where *r* and *z* are the radial and axial observation coordinates, respectively. Eq. (1) is derived and analyzed in terms of error and speed in [4]. To extend Eq. (1) to the more general apodized piston, define an apodization function $q(s)$ the gives the normal velocity of the piston as function of radius s. As noted by [5], the pressure field generated by this apodized piston is synthesized by decomposing the circle of radius *a* into evenly spaced concentric annuli. Fig 1 illustrates this concept. By allowing the number of annuli to approach infinity, the following expression is derived:

$$P(r,z;\omega) = \int_0^a \frac{\partial P_s(r,z;\omega)}{\partial s} q(s)\, ds \tag{2}$$

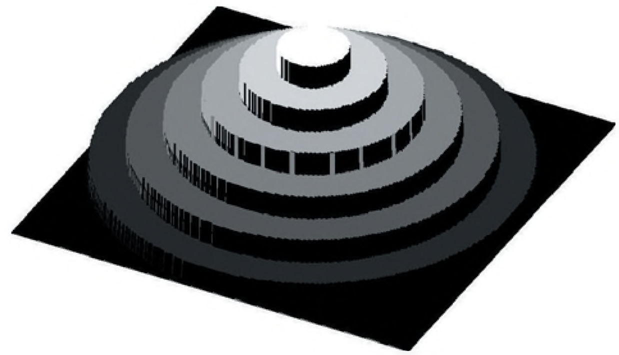

Fig. 1: Schematic illustrating how an apodized piston is decomposed into contributions from concentric annuli.

By assuming that the normal velocity vanishes on the boundary of the piston (*q(a) = 0*), Eq. (2) is integrated by parts, yielding

$$P(r,z;\omega) = -\int_0^a \frac{\partial q}{\partial s} P_s(r,z;\omega)\, ds \tag{3}$$

Substituting Eq. (1) into Eq. (3) yields the final result

$$P_a(r,z;\omega) = -\frac{\rho c}{\pi}\int_0^a q'(s)s \int_0^{\pi} \frac{r\cos\psi - a}{r^2 + a^2 - 2ar\cos\psi} \times \left(e^{-jk\sqrt{r^2+a^2+z^2-2ar\cos\psi}} - e^{-jkz}\right) d\psi\, ds \tag{4}$$

3. METHODS

Eq. (4) is valid for any radial apodization. One choice for the aperture function is

$$q(s) = 1 - (s/a)^n \tag{5}$$

where *n* is a fixed integer. A rigid piston corresponds to *n = 0*, whereas a simply supported piston piston is modeled by *n = 2* [3]. Although Eq. (4) is difficult to evaluate analytically, the double integral can be evaluated numerically via Gauss quadrature or other standard quadrature rules.

4. RESULTS

A reference pressure field generated by a circular radiator of radius 1.5 mm operating at a frequency of 2.5 MHz is evaluated for a parabolic apodization (*n=2*). In order to compare Eq. (4) with the Rayleigh integral, a reference field is evaluated with 160,000 point Gauss quadratue rule on an axially offset 21 by 75 point grid with quarter-wavelength spatial sampling. Figure 2 shows the normalized pressure amplitude. The apodized beam pattern is more spatially bandlimited than a rigid piston beam pattern; in addition, the beam pattern in Fig. 2 lacks the on-axis nulls that characterize the unapodized piston's pressure field.

The Rayleigh-Sommerfeld integral representing the apodized pressure field was also evaluated using the point source method [6]. Both the FNM and point source approaches were implemented in the C programming language and executed on a 3.0 GHz Pentium IV processor running Red Hat Linux. The FNM and point source method are evaluated with varying numbers of quadrature points. The resulting peak normalized errors and computation times are summarized in Tables 1 and 2.

5. DISCUSSION

Unlike the Raylieigh-Sommerfield integral, the FNM embodied in Eq. (4) does not expereince any numerical difficulty near the piston surface. This smooth behavior leads to a more rapid convergence with respect to number of quadrature points. As evinced by Tables 1 and 2, the FNM requires 0.0064 seconds to achieve 10% peak error, whereas the point source approach requires 0.0258 seconds; thus the FNM achieves a speedup by a factor of 4 at 10 % error level. At 1 % error, FNM achieves a speedup factor is about 4.9 relative to the point source approach. Reduced computation times may be possible by implementing the grid-sectoring method described in [4].

A time-domain analog of Eq. (4) can also be obtained for transient excitations. This time-domain expression may prove useful in evaluating scattered and pulse-echo fields generated by imaging transducers.

Table 1. Quadrature points need for specified peak error.

	10 % Error	1% Error
FNM	40	105
Point Source	140	442

Table 2. Computation times need for specified peak error.

	10 % Error	1% Error
FNM	0.0064 s	0.0165 s
Point Source	0.0258 s	0.0809 s

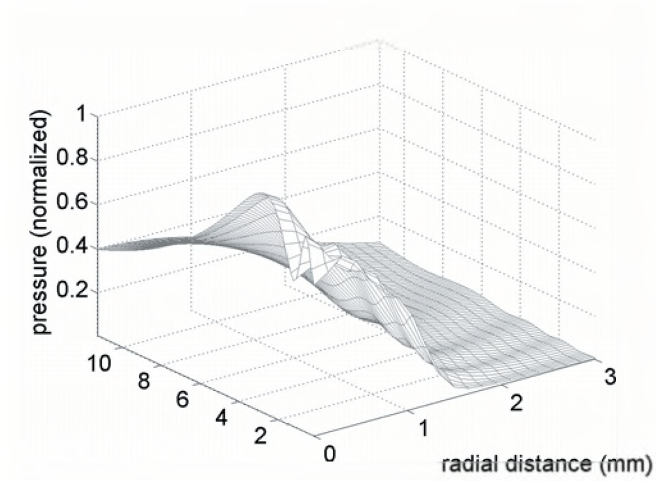

Fig. 2: Normalized pressure amplitude for a parabolic radiator (n=2) with radius a=2.5 mm, or 2.5 acoustic wavelengths. Eq. (3) was evaluated with a 1000 by 1000 Gauss quadrature on an axially offset 21 by 75 point grid with quarter-wavelength spatial sampling.

6. CONCLUSION

A fast nearfield method has been derived for radially apodized circular pistons. Since this method is numerically well-behaved at all observation points, convergence is accelerated as compared to the point-source approach. A speedup on the order of 4 is achieved at 10 and 1 % peak errors.

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