# MULTI-PLANAR ANGULAR SPECTRUM APPROACH APPLIED TO PRESSURE FIELD CALCULATIONS OF SPHERICALLY FOCUSED PISTONS

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## 1. INTRODUCTION

The angular spectrum approach (ASA) is essentially a Green's function method that computes the field by multiplying the source and the Green's function, or propagation operator, in the spectral domain. Some ASA applications use the analytical Fourier transform of Green's function as the propagator [1]-[3]; however, this method suffers from wrap-around error and aliasing. In the far field, large errors are produced because of the truncation of the spatial Green's function. This paper presents a multi-planar angular spectrum approach based on the analysis of the errors. The additional source planes at the path of propagation reformulate the spatial propagator and compensate for the truncation errors.

Most investigations apply the ASA to planar sources, including single transducers and arrays. The ASA for cylindrically curved radiators has also been studied by Wu [4]. This paper will focus on the application of ASA on spherically focused circular pistons.

## 2. THEORY

### 1.1 Angular spectrum approach

The convolution relationship for the waves diffracted from finite apertures is

$$p(x,y,z) = p_0(x,y,z_0) \otimes h(x,y,z), \qquad 1$$

where $p_0(x,y,z_0)$ is the source pressure in plane $z_0$ and $p(x,y,z)$ is the pressure in other planes. For time harmonic wave excitation, the spatial propagator $h(x,y,z)$ can be represented by [5]

$$h(x,y,z) = \frac{\Delta z}{2\pi d^3}(1+jkd)e^{-jkd}, \qquad 2$$

where $k=2\pi/\lambda$ is the spatial wavenumber, $\Delta z = z - z_0$ and $d = \sqrt{x^2+y^2+\Delta z^2}$. If $p(x,y,z)$, $p_0(x,y,z)$, and $h(x,y,z)$ are transformed into spectral domain by 2D FFT, and Eq. 1 becomes

$$P(k_x,k_y,z) = P_0(k_x,k_y,z)H(k_x,k_y,z), \qquad 3$$

where $k_x$, $k_y$, and $k_z$ are angular frequencies of the decomposed plane waves. The wave number $k = \sqrt{k_x^2+k_y^2+k_z^2}$ is constructed from these angular frequency values.

Numerical implementations of these expressions sample the spatial fields at an interval of $\delta$ in both x and y dimensions. Each pressure field input is confined to an $L \times L$ square area and sampled by an $M \times M$ grid. M is preferably an odd integer so that the source is symmetric with respect to the origin. To avoid problems with circular convolution, the source plane is zero padded and enlarged to an $N \times N$ matrix. In general, the accuracy of angular spectrum method is influenced by the spatial sampling rate, the size of the computational grids, and the angular resolution of spectra.

### 1.2. Spatial propagator and propagation distance

For a uniform sampling rate and field size, the errors produced by the ASA vary as a function of depth. The errors generated by the spatial propagator are encountered in two distinct regions. First, as $\Delta z \to 0$ a singularity appears at the origin of $h(x,y,z)$, the steep slope of the signal makes adequate sampling impossible. Thus, the computation of the ASA in the region $\Delta z<1\lambda$-$2\lambda$ consistently produces a relatively large error. Second, as $\Delta z$ increases, the wavefront spreads over larger region. A plane that captures most of the energy in the near field often includes much less energy in the far field. If a significant portion of the power is lost due to truncation, the resulting spectrum is distorted.

## 3. METHODS

### 2.1 Spherically focused piston

Accurate fields of spherically focused pistons can be computed as reference by impulse response algorithm [6] with a high sampling rate. The excitation frequency is 1MHz and acoustic velocity is 1500m/s for the simulation. Lossless media is assumed in these simulations.

### 2.2 Error evaluation

The normalized peak error is defined as the amplitude of the maximum error normalized by the peak pressure in one plane according to the expression

$$\eta_{max} = \frac{\max_{x,y} |p(x,y,z) - p_{ref}(x,y,z)|}{\max_{x,y} |p_{ref}(x,y,z)|}, \qquad 4$$

where $p_{ref}(x,y,z)$ is the reference pressure, and $p(x,y,z)$ is the pressure computed by the ASA. The notation $\max_{x,y}$ indicates that the error is evaluated in a transverse plane.

### 2.3 Multi-planar scheme

An adaptive computing algorithm propagates the fields along z direction and places multiple source planes according to a threshold. First, the errors are evaluated as a function of depth and the position where the error first exceeds the threshold is identified. This determines the location of the next source plane, and the fields in subsequent planes are computed from this source plane until the error threshold is again exceeded. The same process is repeated until all of the source plane locations are determined. The error within a short distance of the first source plane may exceed the threshold, but if the error drops below the threshold after one or two wavelengths, the points adjacent to the initial plane are discarded.

## 4. RESULTS

The multi-planar ASA is demonstrated for a spherically focused piston with diameter 2a=16cm and radius of curvature R=16cm. The normal evaluated at the center of the spherical shell, which is coincident with the z-axis, defines the origin of the Cartesian coordinate system. The computational volume is 16.2cm×16.2cm×32.75cm sampled at an interval of $\delta=\lambda/2=0.075$cm. Each plane is zero-padded to 512×512 points, and then a 2D FFT is evaluated. The adaptive multi-planar algorithm allocates five source planes, and this restricts the peak error to values below 5%. Figure 1 is an illustration of the pressure field in the y=0 plane. This result is obtained from source planes positioned at z=2.25cm, 4.5cm, 11.1cm, 21.45cm and 29.4cm.

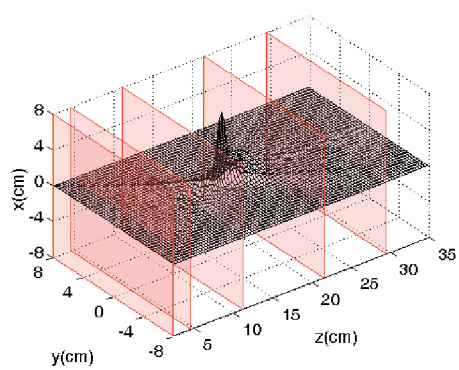


*Figure 1: Illustration of the pressure generated by a spherically focused piston computed with five source planes.*

Figure 2 shows the peak errors as a function of depth. The solid line is computed by the initial source plane at z=2.25cm. With a single source plane (solid line), the error grows rapidly beyond the focus. Shortly thereafter, the error reaches 100%. The dashed line is the multi-planar result computed from five source planes. This result shows that the multi-planar approach maintains an error of 5% or less throughout the computational grid.

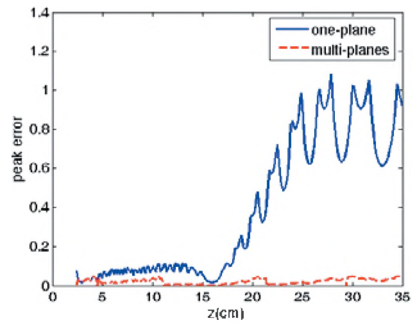


*Figure 2: Peak errors as a function of depth. The solid line represents the error produced by one source plane at z=2.25cm, and the dashed line describes the error obtained with five source planes.*

## 5. DISCUSSION

Since the inadequate truncation of the spatial propagator is one source of error, increasing the size of the sampled grid is expected to reduce the peak error. However, in order to include 90% of the energy in h(x,y,z) at depths of z=10cm and 30cm, the extent of the sampled grid should be 27.2cm and 81.6cm, respectively. This calculation would require an excessive amount of computer memory. The multi-planar scheme reduces numerical errors by efficiently utilizing memory resources without adding a significant computational cost.

## 6. CONCLUSION

The calculations of pressure fields from spherically focused piston using ASA often suffer from truncation errors in the far field. The multi-planar ASA compensates for the error by adding additional source planes and achieves much greater accuracy.

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