A TIME-SPACE DECOMPOSITION METHOD FOR FAST CALCULATIONS OF TRANSIENT PRESSURE FIELDS GENERATED BY ULTRASOUND PHASED ARRAYS

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1. INTRODUCTION

Computing the transient pressure field generated by large phased arrays is helpful in designing ultrasound imaging systems. Methods for computing the transient near-field pressure of a planar aperture include the point-source method [1] and the spatial impulse response (SIR) method [2,3,4]. Recently, a rapid single integral approach has been developed for computing the time domain pressure generated by a baffled circular piston [5]. This solution is applied to a 129 element focused phased array and compared to a similar computation made using Field II [6].

2. THEORY

A time domain solution to the lossless wave equation can be derived subject to an input pulse *v(t)*, which models the uniform normal velocity of the piston. Consider a baffled rigid piston with radius *a* radiating into a homogeneous fluid with density ρ_0 and sound speed *c*. Solving the wave equation in cylindrical coordinates *(r,z)* yields the single-integral solution:

$$p(r,z;t) = \frac{\rho_0 a c}{\pi} \int_0^{\pi} \frac{r\cos\psi - a}{r^2 + a^2 - 2ar\cos\psi} \times \left[v\left(t - \sqrt{r^2 + z^2 + a^2 - 2ar\cos\psi}\right)/c - v(t - z/c) \right] d\psi \quad (1)$$

Although Eq. (1) can be directly evaluated in the time-domain using Gauss quadraure, the computational complextiy is significantly reduced by decoupling the temporal and spatial dependence in the integrand of Eq. (1). Consider a Hanning-weighted pulse *v(t)* with duration *W*, which is decomposed via

$$v(t-\tau) = rect\left(\frac{t-\tau}{W}\right) \sum_{n=1}^{6} f_n(\tau) g_n(t) \quad (2)$$

where the delay τ depends on the spatial coordinates *(r,z)* and the variable of variable of integration ψ. Inserting Eq. (2) into Eq. (1) allows the temporal and spatial dependence to be seperated. The expansion functions in Eq. (2) are computed via trigonometric expansions. This decompositon reduces the number of integrations per observation point from the number of time samples to 6 without introducing any additional error.

3. METHODS

A linear array of 129 circular elements is simulated with pistons of radius *a* = 0.30 mm (half –wavelength) and inter-element spacing *d* = 0.90 mm. Fig. 1 shows the array geometry. A computational grid using quarter-wavelength (0.15 mm) spatial sampling in both the lateral and axial dimensions was employed. The Hanning excitation pulse has a central frequency of 2.5 MHz and duration of 1.2 microseconds. Beam steering and focusing is achieved by application of temporal delays to each element [7]. The total pressure is then synthesized via superposition. Beam steering and focusing are achieved by applying temporal delays. To focus on axis at distance *F* = 50 mm, quadratic delays are employed.

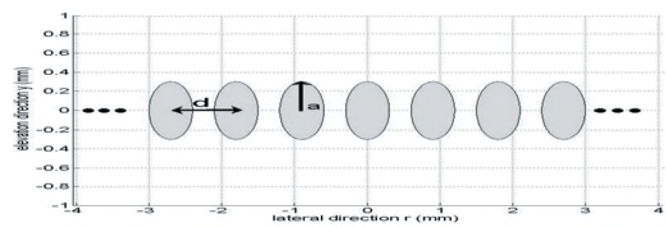

Fig 1: *Densely sampled linear array. The array used in these computations has 129 circular pistons with radius a = 0.30 mm (half –wavelength) and inter-element spacing d = 0.90 mm on a computational grid extending 288 wavelengths in lateral direction by 167 wavelengths in axial direction.*

4. RESULTS

To determine the correct number of quadrature points to apply to the decomposition technique given by Eq. (1), an error analysis is shown in Table 1. As Table 1 shows, Eq, (1) requires 4 Gauss abscissas to achieve a 1 % peak error. Field II, which subdivides the aperture into rectangular sub-elements, requires 484 sub-elements to achieve this error level.

Simulation pressure fields are shown in Fig. 2 at three successive times, where the pressure has been normalized with respect to peak pressure. The total computation time for this array system was 11 minutes. In comparison, similar computations using Field II software[a] [6] took approximately 8 hours to achieve commensurate accuracy. The peak field error is computed relative to a 1000 point

[a] Field II version 2.86 for MATLAB, www.es.oersted.dtu.dk/staff/jaj/field/

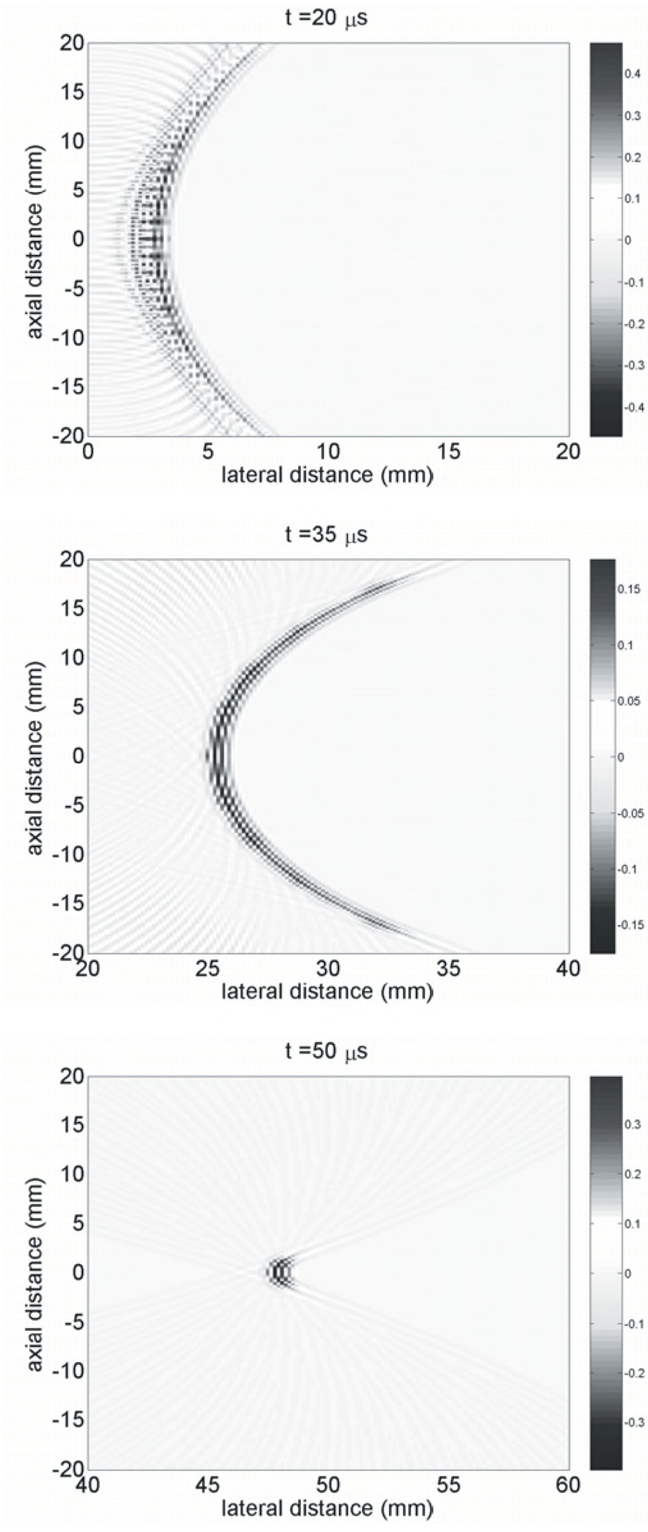

Fig 6: Normalized pressure field at three successive times for the phased array defined in Fig. 1. Pressure is normalized with respect to the peak value On-axis focusing at 50 mm is employed via quadratic time delays.

Table 1. Single-element error analysis.

	10 % Error	1 % Error
Decompositon	3 abscissas	4 abscissas
Field II	12 sub-elements	484 sub-elements

sampling frequency of 32 Hz (compared to Field's 100 reference field; a 4-point quadrature yields a peak error below 1% at all points in the computational grid. Since the present decomposition technique utilizes a temporal MHz sampling), less memory is used.

5. DISCUSSION

Fast and accurate incident pressure field compuations are necessary in several applications. Of particular importance is the interative design of imaging arrays (both 1 and 2D). Array geometry and parameters can be optimized by computing transmitted and pulse-echo pressure fields. Large-scale modeling of wave propagaton and scattering can also benefit from the fast method presented. Time-domain scattering methods, such as generalizations of the fast multipole method (FMM) [8] require incident field data on large, unstructed grids as an input.

6. CONCLUSION

A simulation scheme for pulsed computations with linear phased arrays has been proposed. Unlike previous methods [6], far field and aperture approximations are not used; instead, an exact time-domain solution forms the basis for array simulations, which is accelerated by decomposing the spatial and temporal dependence of the integrand.

REFERENCES

1. Zemanek, J. Beam behavior within nearfield of a vibrating piston. J. Acoust. Soc. Am. 49(1), 181-191.

2. Lockwood, J. C. and J. G. Willette (1973). High-speed method for computing exact solution for pressure variations in nearfield of a baffled piston. . J. Acoust. Soc. Am. 53 (3), 735-741.

3. Oberhettinger, F. (1961).. On transient solutions of the 'baffled piston' problem. Journal of Research of the National Bureau of Standards, *Section B*, 65B, 1-6.

4. Stepanishen, P. R. (1970). Transient radiation from pistons in an infinite planar baffle., J. Acoust. Soc. Am. 49 (5), 1629-1638.

5. Kelly, J. F. and R. J. McGough (2005, submitted). A time-space decomposition method for calculating the near field pressure generated by a pulsed circular piston. IEEE Trans. Ultrason. Ferroelect. Freq. Contr.

6. Jensen, J. A. and N. B. Svendsen (1992). Calculation of pressure fields from arbitrarily shaped, apodized, and excited ultrasound transducers. IEEE Trans. Ultrason. Ferroelect Freq. Contr., 39 (2) , 262–267.

7. Von Ramm, O. T. and S. W. Smith (1983). Beam steering with linear arrays. IEEE Trans. Biomed. Eng. 30 (8), 438-52.

8. Ergin, A. A., B. Shanker, and E. Michielssen (1999). The plane-wave time-domain algorithm for the fast analysis of transient wave phenomena. IEEE Antennas and Propagation Magazine, 41(4), 39-51.

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