INVESTIGATING THE SOUND OF AN AFRICAN THUMB PIANO (KALIMBA)

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1. INTRODUCTION

The African thumb piano, or kalimba (also called by other names) is an unusual percussion instrument consisting of a number of thin metal blades (keys) mounted on a soundbox or soundboard. (See Figure 1) The keys are mounted with different lengths to produce different notes, and they are typically struck at the end by the performer's thumbs while the soundbox is held by the fingers.

The individual notes of a small kalimba were recorded and frequency-analysed to determine the overtone structure. The notes have a definite pitch, a sharp attack, and a rapid decay; the quality of the note could be described as a "sproing". The dominant tones of a note are: a strong fundamental (f_0) and a single anharmonic overtone (f_1) whose frequency ratio f_1/f_0 is about 5.3-5.9. (There are also higher-frequency, less significant, overtones.)

Using these facts, one can construct a synthetic kalimba waveform from the sum of two exponentially damped sinusoids. By aurally comparing true and synthetic waveforms, it is possible to adjust the few model parameters to produce a realistic-sounding synthetic waveform.

Unlike the modes of a vibrating string or organ pipe, the modes of vibration of a thin bar produce anharmonic overtones (i.e. non-integer-multiples), which provide a mild harshness to the tone quality owing to the discordant musical intervals produced. The principal thrust of this investigation is to investigate the overtone structure of the kalimba notes, and to understand how the key-mounting method affects the f_1/f_0 ratio. Although this task is incomplete, initial results suggest that the overtone structure on this particular kalimba is sensitive to the details of the mounting geometry. Photographs and sound files associated with this project can be found at the web page:

http://homepage.mac.com/chapmandave/Kalimba/

Figure 1: The small kalimba used for the measurements.

2. MEASUREMENT

The kalimba notes were individually recorded and analysed on an Apple Macintosh iMac Intel Core Duo computer, using the built-in microphone and A/D converter, controlled by the freeware audio software AUDACITY. It was decided that this arrangement was sufficient to study the overtone structure of the kalimba, and that a calibrated microphone was unnecessary; by the same token, no special means of exciting the notes was devised: some care was taken to produce uniformly loud notes with the thumbs as one would in a typical performance.

The files produced were common .wav audio files, which were imported into MATHEMATICA 5.2 for analysis. The imported waveforms were plotted and replayed using standard functions. The frequency spectra of the waveforms were computed using the `Fourier` function (with no averaging) and plotted for inspection. Using the cursor within the MATHEMATICA plot, f_0 and f_1 were measured for all notes.

Referring back to Figure 1, notice that the keys have a peculiar mounting arrangement: essentially the keys are thin bars that are free at one end (the striking end) and clamped at the opposite end; however, partway along from the clamped end, the keys are supported by a fulcrum and held in place by a restraining bar. Through simple experimentation, it was discovered that the low-amplitude motion of the short length between the fulcrum and the bar is not inconsequential and should be taken into account. (If one presses on this portion while a key is struck, the sound is noticeably damped.) By the same token, the portion between the bar and the fixed end was considered to be entirely clamped, and irrelevant to the acoustics. Accordingly, the individual key lengths free-fulcrum (a) and fulcrum-bar (b) were measured.

Nominally (by comparing with a piano) the kalimba notes form a pentatonic scale starting at the D in the third octave: D3 E3 G#3 A#3 D4. Table 1 shows the nominal note name, nominal note frequency, lengths $a$and b, ratio a / b, fundamental frequency f_0, overtone frequency f_1, and frequency ratio f_1/f_0 for all the notes. Figure 2 shows the waveform of the D3 note and Figure 3 shows the corresponding spectrum. In Table 1, note that the ratio f_1/f_0 is not fixed, but is different for each note, and systematically decreases with the a / b ratio (although G#3 is an anomaly).

Table 1. Kalimba note measurements.

note[a]	$f^{(b)}$ (Hz)	a (cm)	b (cm)	a/b	f_0 (Hz)	f_1 (Hz)	f_1/f_0
D3	293.7	5.4	1.4	3.9	304	1780	5.86
E3	329.6	5.0	1.3	3.8	336	1890	5.63
G#3	415.3	4.6	1.4	3.3	415	2390	5.76
A#3	466.2	4.2	1.3	3.2	462	2590	5.61
D4	587.3	3.7	1.3	2.8	581	3080	5.30

[a] the nominal note, i.e. the piano note closest to the kalimba note
[b] the frequency of the nominal note

3. SIMULATION

By inspecting both the measured waveforms and the spectra of the notes, it was decided that a fair representation of the note would be the sum of a pair of sine waves at frequencies f_1 and f_0, with exponentially decaying envelopes. The frequency parameters are estimated from the spectra; the decay time of the fundamental is estimated from the waveform (and validated by listening). This leaves the overtone/fundamental amplitude ratio and the overtone decay time to be determined. It was assumed that the overtone would decay quicker than the fundamental, and this is supported by detailed examination of the waveform, although it is difficult to precisely estimate this decay time. (Filtering might help.) The same comment holds for the overtone amplitude, although some idea of the overtone amplitude ratio can be drawn from the spectrum. The overtone amplitude ratios and decay times were estimated by guesswork and listening. Casual listening by non-experts confirms that the synthetic notes are a fair representation of the real tones. Work continues on this aspect, with a view to determining the model parameters objectively.

4. THEORY

Lord Rayleigh (1896) performed the classic analysis of the modes of a thin vibrating bar. The wave equation for transverse vibrations is a fourth-order linear differential equation whose solutions are combinations of sin, cos, sinh, and cosh functions. Generally, the overtone structure is anharmonic (a fact that is underappreciated) with the precise spacing determined by boundary conditions on the displacement and its first three spatial derivatives. Rayleigh considered the cases of a free end, a clamped end, and a supported end. When the appropriate boundary conditions are applied, the determination of the frequencies of vibration reduces to finding the zeros of a transcendental equation, which is tackled numerically. For the clamped-free bar, which is the closest simple system to the kalimba keys, Rayleigh found $f_1/f_0 = 6.2669$.

Applied to the kalimba in question, the analysis is somewhat complicated by the additional support in the middle, but this will be accommodated by considering the key to be composed of two bars on either side of the fulcrum (one clamped at the end, the other free), with matching conditions where they join. The hypothesis is that the ratio f_1/f_0 for each key is governed by the a / b ratio of the key, and an explanation for the reduced f_1/f_0 ratio will be sought.

The modeling of this vibrating system and the numerical determination of overtone frequencies is ongoing work.

REFERENCES

Rayleigh, John William Strutt (1896). *The Theory of Sound, Volume One*, Chapter 8 (Dover Publications Inc., New York)

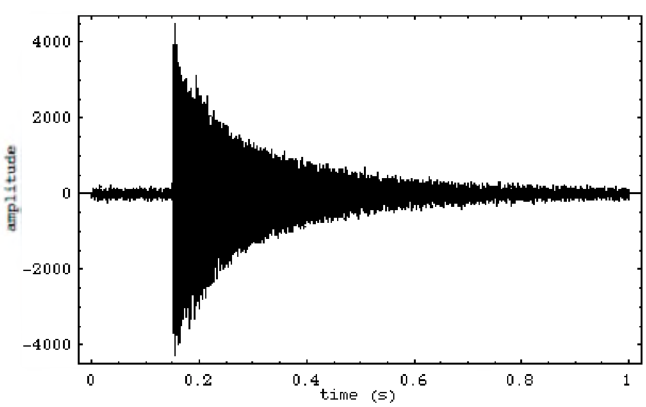

Figure 2: Waveform of note D3. Notice the sharp attack and exponential decay of the waveform.

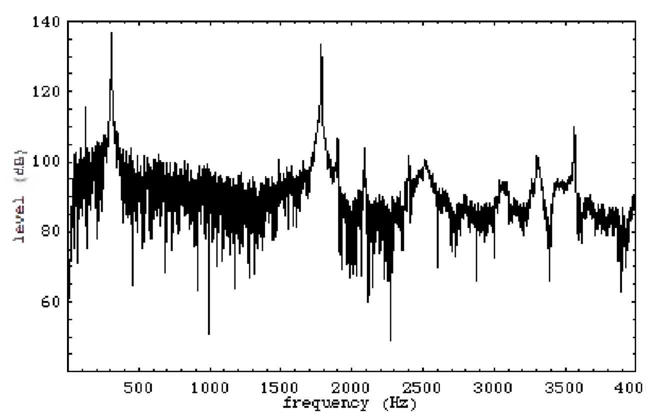

Figure 3: Spectrum of note D3. Notice the fundamental at 304 Hz, the first overtone at 1780 Hz, and a second overtone at 3560 Hz (highly attenuated).